Logic, Probability and Inference:
Adjusting the Methodology to a New Paradigm

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Abstract

The psychological investigation of reasoning has adopted a new paradigm in which it is assumed that in their daily life individuals normally reason under uncertainty even when they engage in a deductive activity. In this approach the Bayesian model is substituted for classical bi-valued logic as a normative reference in the experimental investigation of human reasoning. This raises new conceptual and methodological questions. Here two levels of study are distinguished: An epistemic level, at which the underlying formal model is built on trivalent logics, and a meta-epistemic level, at which the underlying formal model is probabilistic and yields belief intervals. For the first level the difficulty in investigating the validity of inferences is underscored; this is due to the existence of a plurality of definitions of validity in a non-bivalued framework. The solution that is proposed is to identify, among the theoretical trivalent connectives, those which coincide with natural language connectives. At the second level, two objectives are distinguished, both requiring a novel methodology. The first one consists in investigating individuals' compliance with probabilistic validity (following the notion of *p-validity* developed by Adams, 1998) and the second one in examining the *coherence* of individuals' probabilistic evaluations (in de Finetti's sense). Several arguments are put forward in favor of the Finettian coherence method.

The new reasoning paradigm

Psychologists who investigate human reasoning define human rationality by using formal models that predict individuals' optimal behavior with respect to their objectives. In particular, some formal models have generally been considered as norms of reference for rational inference. Traditionally psychologists have assumed that different psychological processes correspond to the different formal domains of reasoning: decision making, probability judgment, inductive reasoning, and deductive reasoning. Three main formal models have been used as reference to study these domains: subjective expected utility for decision making, the Bayesian model for probability judgment and induction, and binary classical logic for deduction. It must be kept in mind that the choice of a specific model, which is guided by theoretical options, may have deep methodological consequences for the experimental work.

As far as deductive reasoning is concerned, several periods and lines of research can be identified, which reflect the status given to classical logic by psychologists. In the early days, research on reasoning was focused on individuals' formal reasoning capabilities and the unique reference was Aristotle's syllogistic. This is the case of Binet (1902) and James (1908). Piaget (Inhelder & Piaget, 1955) extended the model of reference to propositional logic but again regarded the adult human being as an implicit logician. In sum, theorists chose that portion of classical logic known to them both as a descriptive and a normative system of reference. However, the work of Wason & Johnson-Laird (1972), revealed that although individuals exhibit some logical competence overall, their answers to a variety of deductive tasks are too often at variance with logic for the Piagetian position to be defensible. At about the same time the results of the "heuristics and biases school" (Kahneman, Slovic & Tversky, 1982) highlighted an apparent gap between individuals' probabilistic judgment and the "Bayesian" norm, from which a "pessimistic" view on human rationallity ensued. Even though the introduction of an approach to human reasoning based on linguistic pragmatics (Hilton, 1995; Politzer, 1986, 2004; Politzer & Macchi, 2000) helps explain a wide variety of reasoning errors, its is clear that classical logic does not adequately describe people's deductive performance. In the 1980s and 1990s, the bulk of the theoretical debate about deduction revolved around the nature of the representation and processing of the input information, assumed to be syntactic and rule-governed by some researchers (Braine and O'Brien, 1998; Rips, 1994) or semantic and based on mental models by others (Johnson-Laird & Byrne, 1991). The research agenda was to explain both individual's competence and its limitations while keeping a classical view of the normative system of reference, that is, a logic whose inferences are truth-preserving, which is monotonic and whose premises are certain. Meanwhile two domains of research started to have a decisive impact on the investigation of deduction. One, researchers in Artificial Intelligence were studying topics or phenomena hitherto disregarded by psychologists, such as abduction or defeasibility (that is, the retraction of a conclusion upon learning a new piece of information). Two, the development of research on decision making using probability theory as its main tool contributed to bring uncertainty to the forefront. Both domains aim to study thinking and reasoning in real life, where people exploit their huge knowledge bases, premises are uncertain, conclusions defeasible, and reasoning as a whole is goal-oriented. In the 1990s it became apparent that the scope of research on deductive reasoning was too narrow, if not misdirected. A call for a change in the normative system of reference was made by Oaksford & Chater (1995, 2001) who argued that logic is inadequate to account for performance in reasoning tasks because reasoners use their everyday uncertain reasoning strategies, whose nature is probabilistic, so that individuals' inferential activity should no longer be assessed only with regard to logical axioms; rather, it should be assessed in reference to probability theory. This view has the advantage that it integrates the studies of reasoning and of judgment and decision making into a unified domain of research. Even though other formalisms are possible candidates for psychological modelling (see Politzer & Bonnefon, 2010) probability theory has been adopted by most researchers who endorse this new approach. A majority, following Oaksford & Chater (2007) adopt Bayesianism as a normative model[1] (although the need to choose a normative system has been questioned, see Evans, 2012).

This new approach to the study of reasoning has been regarded as a paradigm shift (Evans, 2012) and called the *new paradigm psychology of reasoning* (Over, 2009). It incorporates two major assumptions.

1. The indicative conditional[2] of natural language *if A then C* does not necessarily coincide with the material conditional of formal logic which specifies a strong relation between A and C (A entails C). Rather it expresses a link (possibly weak) between A

and C. Consequently the valid inferences involving the material conditional that are paradoxical for common sense are not always correct.[3]

2. The conclusion of a deductive argument may carry some uncertainty inherited from the premises' uncertainty (George, 1995, 1997; Liu, Lo,& Wu, 1996; Stevenson & Over, 2011). The uncertainty is represented by an additional truth value and the logic of reference is a tri-valued logic (Baratgin, Over & Politzer, 2014). The uncertainty is revealed in an explicit way by probabilistic inferences that can be analyzed in the light of Bayesian coherence (Pfeifer & Kleiter, 2006).

In the new paradigm the normative model of binary logic is replaced with a Bayesian model (considered as providing a "probabilistic logic") to study the uncertain deductive inferences which people carry out in daily life.

Of course, there are various conceptions of Bayesianism. The philosophical, logical, linguistic, economic literatures, and also the artificial intelligence literature offer a wide variety and the choice of one or the other form for normative or descriptive purposes has important consequences (Baratgin, 2002; Baratgin & Politzer, 2006 ; 2007; Elqayam & Evans, 2013).

In what follows, a relatively radical choice is made, namely the adoption of subjective Bayesian probability (de Finetti, 1937; Ramsey, 1926/1990; Savage, 1954). There are several reasons for this choice. One, this theory provides a criterion of rationality that can be used experimentally in an effective manner (see below). Two, its conception of probability as dependent on the individual's state of knowledge establishes the investigation of probability judgment as an interdisciplinary object open to psychological investigation, as de Finetti (1957, 1974) himself suggested. Three, de Finetti (1930) explicitly stated the hypothesis that reasoning is naturally probabilistic[4]:

> The calculation of the probabilities is the logic of the probable. Formal logic teaches us to deduce the truth or falsity of certain consequences of the truth or falsity of certain premises; likewise the calculation of probabilities teaches us to deduce the greater or lesser likelihood or probability of certain consequences of the greater or lesser likelihood or probability of certain premises (p. 261-262, our translation).

Even though the subjective interpretation of probability was defended by several scientists right from the beginning of probability theory, two authors can be regarded as the "father founders" of subjective Bayesianism: Bruno de Finetti and Frank P. Ramsey created independently and virtually simultaneously at the end of the 1920s a theory founded upon a subjective, or personal, interpretation of probability. Some time later Leonard L. Savage proposed a related approach that generalizes to decision theory. However, de Finetti's theory differs from it by at least three specific characteristics:

1. "Radical" subjectivism.[5] De Finetti emphatically rejects any conception of probability that is not subjective, meaning that probability is measured by the degree of belief of a given individual (at a given instant). From his point of view probability is in all cases the expression of a subjective opinion, be it defined as the limit of a frequency, or a proportion dictated by the logical analysis of the possible cases. Many other "subjective" Bayesian theorists accept, under some conditions, the notion of physical probability viewed as irreducible (applicable when probability expresses the intrinsic property of a phenomenon all the factors of which are known or regular), and also the notion of ontological probability or objective chance (see Good, 1976 ; Jeffrey, 1976 ; Lewis, 1980). Ramsey himself defends a dualistic position for probability (see the note "chance" in Ramsey (1926/1990) and Galavotti's analysis (1995, 1997, 1998).

2. De Finetti's theory is also specific by its direct method of evaluation of probability. For Ramsey (1926/1990) probabilities and utilities are simultaneously revealed to the individual by a system of axioms that define coherent preferences. For Savage (1954) it is the agents' choices that allow them to attribute numerical values to their beliefs. In contrast, for de Finetti a person can directly assess the probability of occurrence of a particular event by considering how much she is willing to bet in favor of the realization of this event. The Dutchbook procedure (that is, an assessment resulting in a sure loss in case of a series of bets) is the only criterion of rationality together with the operational method of betting that allows to justify the notion of coherence. It amounts to saying that rational individuals cannot contradict themselves. This process of direct evaluation is essential from an experimental point of view because participants in experiments are explicitly required to directly express a probability.

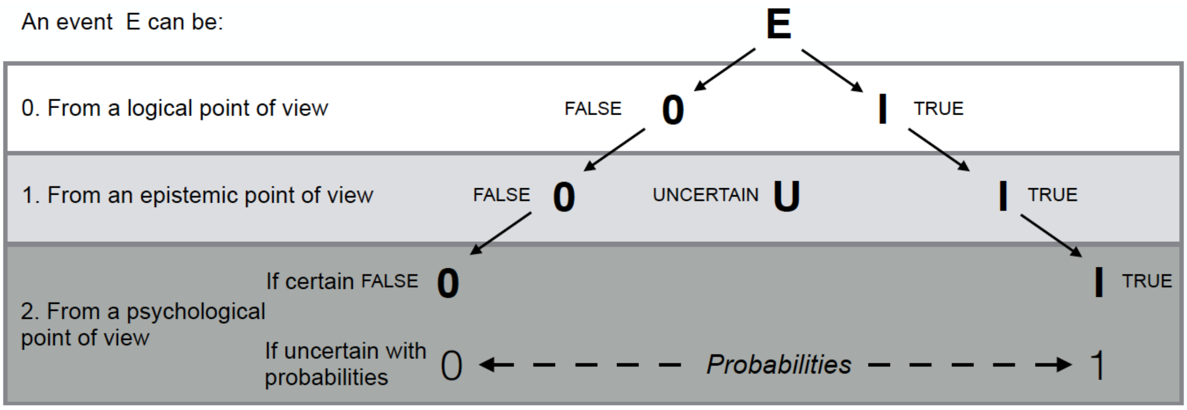
3. Lastly, based on the notion of exchangeable events (meaning that their probability is invariant under permutation, see de Finetti 1931b, 1937, 1979b) and the subjective

conception of probability, it is possible to reproduce all the classical properties and theorems that are known in the frequentist interpretation of probability. The evaluation of a probability, the unique result of the quantification of the judgment of likelihood attributed to an isolated event given by a particular individual on the basis of his or her own knowledge coincides in every respect with what experimenters studying probability judgment ask of participants. It seems that the Finetian model can naturally be associated with the psychological investigation as they have the same objective (Baratgin, in press).

Two levels of analysis: a dual system

In the old paradigm the validity of the arguments presented to the participants was dictated by formal logic independently of the meaning of the propositions or events that constitute the arguments. The experimental study relied on the acceptance of the truth of the premises by the participant. The participants' inference was revealed by their characterization of the conclusion presented to them in terms of "true", "false", (or "one cannot know"), or by the production of their own conclusion. The main objective was to determine whether the formulas (in particular the connectives for propositional logic) were correctly interpreted (that is, in agreement with the truth tables of classical logic) and whether participants complied with valid arguments of formal logic. In the new paradigm this methodology is not always applicable because in the first place one must consider which of the individual's "levels of knowledge" is being investigated. The old paradigm considers only the logical level, which is what de Finetti (1980) calls the objective level of knowledge, or level 0, where an event is either true or false in an absolute way. Even if this level is ideal from an axiomatic point of view, it is practically sterile because it has no other use than the enumeration, and the exposition in a different shape, of already known events (de Finetti, 2006). In the new paradigm, it is useful to consider the two supplementary levels of knowledge of an event defined by de Finetti (1980), both of which are subjective (Figure 1).

Figure 1. The three levels of knowledge (de Finetti, 1980).



Level 1, the elementary level of knowledge, is the level of the individual's beliefs about the events. An event has an epistemic and subjective import. It concerns a specific object or phenomenon defined by its own characteristics known to the individual. An event is always conditioned on the individual's personal state of knowledge. For de Finetti the value *true* attributed to an event corresponds to its realization and the value *false* to the absence of its realization. From now on, we will qualify an event by *true* or *false* in this sense. Now, very frequently we are uncertain about de la realization of an event. This is why de Finetti defends the notion of a three-valued logic to define the (conditional) events. A conditional event may be true, false, or *doubtful*. This "doubtful" value reflects a transitory epistemic state of uncertainty which depends on the individual who does not know the event's truth value at a given instant. This third value should not be compared ordinally with the absolute values *true* or *false* of binary logic; rather is it to be conceived of as a "null" or "void" value. This leads to a three-valued logic "provisional and superimposed" on ordinary bi-valued logic (de Finetti, 1936, p. 39). The individual is also assumed to be capable of executing logical operations on the propositions associated to the events. As already mentioned, an event is always conditionnal; when necessary, following de Finetti, we will call it a "tri-event" or a "conditional event" (noted C/A). A (conditional) event expresses a link between the occurrence of two other events: the event A and the event C. This defines a weakened form of the material conditional (for which A entails C). The conditional event is true when A and C are true, false when A is true and C is false, and null when A is false independently of the value of C. This constitutes the framework of a tri-valued logic of which the system outlined in Baratgin, Over & Politzer (2014) constitutes a possible construal. The principal property of this elementary system is the fundamental relation of the conditional event defined by de Finetti (1936/1995) written here using his own notations:

If A then C = C/A = C&A/ A (1)

Level 2, the *meta-epistemic* level of knowledge, concerns the *degrees* of belief about the event which are subjective additive probabilities. It is assumed that during the transitory state of uncertainty the individual is capable of producing a degree of belief (always dependent on his or her state of knowledge) about the doubtful event (which could lead to a multiple-valued logic--in fact an infinity of values). The individual's judgments of probability apply in particular to the conclusion of arguments knowing the subjective probability of the premises. As far as the conditional event is concerned, the principal property of this level is that the probability of an indicative conditional of natural language, *if A then C,* is equal to the conditional probabilty of C given A, P(C/A), and not to the probability of the material conditional as was assumed in the old paradigm (which is equivalent to 1 - P(A &¬C)). This property reads:

P(if A then C) = P(C/A) (2)

This equation is supported by various arguments, philosophical and logical (see Adams, 1998; Edgington, 1995 for a review), and psychological (Cruz & Oberauer, 2014; Evans & Over, 2004; Oaksford & Chater, 2007, 2009; Pfeifer & Kleiter, 2010).

The relationship between the two levels of knowledge

The passage from the elementary level where the individual has an opinion about an event characterizable by three truth values to a meta-knowledge level where the individual evaluates the event's probability quantitatively is an open field of study for psychologists --including the problem of the transformation of degrees of belief into quantitative values within the second level. Psychologists generally implicitly assume that these values are additive because, following the adoption of the Bayesian framework, they are committed to the endorsement of this property. De Finetti offers the outline of an account of the relationship between the two levels. At a first stage the intuition of the probability of occurrence of an event is qualitative and can be positioned on an ordinal scale; it is therefore susceptible of a comparison with another event. At a second stage a quantitative evaluation may follow, but it is constrained by the rules of additive probabilities which define the static coherence (see Baratgin & Politzer, 2006). The betting schema (de Finetti, 1937/1974) reveals the trivalent character of an event and allows to obtain a "quantitative, numerical, definition of the degree of probability attributed by a given individual to a given event" (p. 101). The relations (1) and (2) can be obtained directly by the betting schema:

> « A tri-event corresponds, in contrast, to a bet whose validity is subordinate to some conditions which must be verified. One can bet, for example, on the victory of one of the competitors in a race which ought to take place tomorrow; if one understands that the bet is totally lost if this event does not take place, one is in the first case; one is in the second case, if one establishes that the bet is null and without effect if the race can not take place, if the competitor in question is not able to participate, or in any other eventually whatever […] The probability (subjective) attributed by the individual O to an event A (in general tri-event) is the price p at which he considers it equitable to exchange a sum *pS* for a sum S, the possession of which is conditioned by the verification of A (in the case of tri-events it is necessary to specify again: the payment of the stake *pS* is conditional on the arrival of the "hypothesis" A). » de Finetti (1936/1995, p.185-186).

The relation (2) can also be directly established based on the consideration of the three possible values of an event. The probability of the event is the probability that the event is true given that it is defined (de Finetti, 1936).

Besides these formal foundations, the relation (2) has a psychological justification. It can be interpreted as the application of the procedure devised by Ramsey, called the "Ramsey test" in the philosophical literature:

> « If two people are arguing ‘If p will q ?’ and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q ; so that in a sense ‘If p, q’ and ‘If p, not-q’ are contradictory. We say they are fixing their degrees of belief in q given p. If p turns out false, these degrees of belief are rendered *void.* If either party believes not-p for certain, the question ceases to mean anything to him except as a question about what follows from certain laws or hypotheses » (Ramsey, 1929/1990 p.143).

Assuming that at the second level the degrees of belief are additive subjective probabilities (which was proposed by Ramsey, 1929/1990) the test can be interpreted as the description of the psychological process of moving from level 1 to level 2 (which is at the heart of the suppositional theory of probability, see Evans, Handley & Over, 2003; Evans & Over, 2004; Handley, Evans & Thompson, 2006).

The individual imagines the antecedent or makes the supposition[6] that p, then evaluates the probability of the consequent q[7]. Ramsey (1929/1990 p.143) does not go through the definition of the conditional event as in (1); rather he describes a dynamic process of belief revision. A conditional is accepted with respect to an initial state of knowledge, if this knowledge revised by the antecedent p (which is here considered as the revision message) validates the consequent.

The new paradigm has focused primarily on the indicative conditional in natural language. In doing so, it has addressed the two problems of the comprehension of the conditional and the evaluation of its probability as a conditional probability. These are two different objects of study that have not been clearly distinguished so far. It seems important to distinguish de Finetti's conditional event from the probabilty of this event. It is the latter that has been studied the most through Adams' "conditional probability". However, we believe that it is important for the experimental studies to define a specific methodology for each of these two levels of representation of belief.

Comparing the methodology for studying the two upper levels of knowledge

Studying individuals' inferences at the level of the events and at the level of their probability may require different methodologies. For the first level the methodology is close to the traditional bi-valued case provided a norm of inferential validity has been chosen and a tri-valent system has been precisely defined. For the second level, there are two possibilities. One is to choose a general notion of *probabilistic* validity and, like at the first level, the experimenter observes whether participants' inferences comply with validity. The other is to exploit the informational richness of the domain and to adopt identically the methodology used to study probability judgment, which is based on the examination of the coherence of participants' evaluations. We detail this below.

The conditional event

Tri-valued logics and validity. At the elementary level, by giving up classical logic the new formalism widens the bi-valued semantic framework (T=true, F=false) to adopt a tri-valued one (T, F, U), where U admits of different interpretations (for a review, see Ciucci & Dubois, 2012). As is well known, in trivalent logic the concept of tautology (like its dual concept, contradiction) is equivocal, contrary to binary logic (in which a sentence is a tautology if its truth value is *true* whatever the truth value of its constituents). Similarly, the validity of an argument in binary logic is defined by the preservation of truth from premises to conclusion (which is equivalent to the preservation of non-falsity). In trivalent logic the definition of tautology depends on the choice of the *designated* value(s) and the definition of contradiction depends on the *antidesignated* value(s) (see Rescher, 1969, p. 66-71). In brief, before any experimental investigation, it is necessary to ponder the definition of a tautology, a contradiction, and a valid consequence. One guideline may help make a choice: it seems desirable that the new formalism allow to explain the robust results of the old paradigm as a limiting case. This leads to three desiderata.

1. The inferences that are valid in the traditional sense and widely accepted by reasoners should be retained as valid by the new definition. For example, it is well established that virtually everyone endorses the conclusion of Modus Ponens and a clear majority endorses the conclusion of Modus Tollens (for a recent review, see Manktelow, 2012).

2. Many studies have shown that people reason nonmonotonically (Bonnefon & Hilton, 2002; Byrne, 1989; Pelletier et Elio, 1997; Politzer, 2005; Stenning & van Lambalgen, 2008; Stevenson & Over, 1995; for a review see Politzer & Bourmaud, 2002). In particular the eight axioms of system P (Kraus, Lehmann & Magidor, 1990; Lehman & Magidor 1992), namely, Reflexivity, Left Equivalence, Right Weakening~~,~~ Cut, Cautious Monotonicity, Equivalence, And, and Or seem to be endorsed by a majority of reasoners (see Benferhat, Bonnefon & Da Silva Neves, 2004, 2005; Da Silva Neves, Bonnefon & Raufaste, 2002). If these results are confirmed, it is desirable that these axioms constitute valid inferences according to the next definition.

3. The definition should not include the arguments that are not endorsed by the majority of the reasoners. This is the case, for instance, of the two "paradoxes" of the conditional: the negated antecedent and the affirmed consequent (for the latter, see Bonnefon & Politzer, 2011).

In the literature, there are at least four possible definitions of validity that have been applied to tri-valued logics.

V_T. T is the designated value. Validity is defined by the preservation of truth from the premises to the conclusion and the preservation of falsity from the conclusion to the premises. It is also the definition of validity generally adopted by tri-valued logics.[8] Formally, it coincides with Kleene's trivalent system.[9] Egré & Cozic (in press) describe the main inference schemas of de Finetti's tri-valent logic valid under this definition. The definition of tautology (and contradiction) coincides with the classical definition of bi-valued logic.

V_{nF}. The designated values are T and U. An argument is valid if it preserves non-falsity from the premises to the conclusion. It is necessary that no non-false premise leads to a false conclusion.[10]

V_{TnF}. This definition is the conjunction of the two previous definitions. There is preservation of truth from premises to conclusion along with preservation of non-falsity from premises to conclusion. It is impossible that the conclusion be false without at least one premise being false. Taking the following order for the truth values: F < U < T, V_{TnF} prescribes that the conclusion cannot have a truth value weaker than the minimum of the truth values of the premises. This definition of validity is adopted by Blamey (2001) and McDermott (1996).[11]

V_U. A validity has been proposed under the name of "correct inference" by Hailperin (1996) for his "suppositional logic", which is identical to de Finetti's with "indeterminate" as a third value, and F < U < T. Hailperin defines validity as the preservation of U-validity from premises to conclusion.[12]

Validities V_T and V_{TnF} are not convenient from a psychological point of view because they do not lead to the recognition Modus Ponens as a valid argument. V_T , V_{NF} and V_{TnF} do not respect all the axioms of system P.[13] So, validity V_U seems the most appropriate to specify de Finetti's system. However, even if it is necessary to define a validity to start an experimental work on inference schemas, it is far from sufficient. Although de Finetti was a precursor, he is not the only theorist to have defined the natural language conditional as a conditional event. The formal literature in philosophy, linguistics, and AI offers other interpretations of the conditional that can follow a different semantics while remaining trivalent (for a review see Baratgin, Politzer, Over & Takahashi, in preparation).[14] This is why our first experimental objective was to identify, among different trivalent logical systems that incorporate a conditional event, a system that is the closest possible description of the interpretation of natural language connectives (Baratgin, Over & Politzer, 2014). We have considered three possible conditional events and nine possible trivalent systems.

Preliminary results. We have carried out a series of experiments in which participants had to construct three-valued truth tables for the main connectives (negation, conjunction, disjunction, conditional, and the material conditional worded as not(A and not-C). The material consisted of chips defined by their color and shape. The color could be clearly black, or clearly white, or uncertain, and the shape clearly round, or clearly square, or uncertain. Two alternative scenarios allowed the assertion of a sentence or of a conditional bet on the same sentence. The main results are summarized below (see Baratgin, Over & Politzer, 2013 for the indicative conditional).

1. For all the connectives considered, participants' answers in the two scenarios are very close. They treated the questions relative to the truth, the falsity or the uncertainty of an assertion in the same manner that they treated the questions relative to winning or losing, or calling off a bet on that sentence; and this applies in particular to the conditional. These results confirm and generalize the observation made by Politzer, Over & Baratgin (2010) that a majority of participants judged that, when the antecedent A of a conditional *if A then C* is false, the sentence is neither true nor false and when A does not occur a bet on this conditional is neither won nor lost,[15] suggesting that for the speech acts of conditional assertion and conditional bet there is a similar, trivalent, interpretation of the conditional sentence.

2. Considering first only the four cells of the classical bi-valued truth tables (that is, restricting ourselves to the two values T and F (or win and lose) participants' answers coincided with the traditional tables for negation, conjunction, and, to a lesser extent, disjunction. In contrast, answers for the conditional were distributed essentially over two tables coinciding with the conditional and the conjunction. A conjunctive interpretation for the conditional with abstract material has already been observed (Girotto & Johnson-Laird, 2004) and our results confirm this with an even higher rate.[16]

3. Next, for the various connectives under study, we have considered all 9 cells of the three-valued table defined by T, F, U (or win, lose, void) and we have analyzed each participant's truth table and compared it with the tables available in the literature (see for example Rescher, 1969). This analysis has produced three main results.

(a) Most tables produced by participants can be classified among a limited number of truth tables belonging to the 9 systems mentioned earlier. This is remarkable, as there are theoretically 3 (values) power 5 (cells with one or two U marginal values) =243 possible truth tables for each connective. In other words, participants treat uncertainty in a systematic manner.

(b) For negation, almost all participants have produced the table of *involutive* negation (in which the negations of U, T and F are U, F, and T, respectively). For the other connectives the majority of the answers coincide with tables that belong to de Finetti's logic. This applies both to the assertion and the betting conditions.
(c) Most participants have given different tables for the conditional event and the material conditional. This result, obtained with a disambiguated form of the material conditional and using three truth values, extends the observations made in the old paradigm showing that lay reasoners do not interpret the natural language conditional as a material conditional; rather the most frequent interpretation is de Finetti's conditional event.

In conclusion, these first experimental data point to de Finetti's trivalent logic as an adequate descriptive system at the first level of knowledge. This system seems to offer a good description of human evaluation of logical relations under uncertainty. This applies in particular to the conditional event and explains an emblematic result of the old paradigm, namely the "defective" truth table. However, these results have been obtained using a third truth value explicitly defined as a physical uncertainty bearing upon the characteristics of an object--here a visual uncertainty. The limitation is that we still need to understand how individuals construe this third value. Further work is necessary to address this essential question.

Conditional probability

At the second level of knowledge, the shift from formal logic to Bayesian probability can be done in two ways.

Probabilistic validity. One way consists of replacing the traditional notion of logical validity of an argument built on truth preservation, viz. , *it is impossible for the premises of a valid argument to be true when the conclusion is false,* by a notion of probability preservation. There are various choices (see Adams, 1996) such as, for instance *it is impossible for the probability of the premises of a valid argument to be non-null when the probability of the conclusion is null.* Other definitions of probabilistic validity are focused on the probability of the conclusion, given the probability of the premises. They are concerned with the possibility of warranting a sufficiently high value (typically a threshold > 1/2) for the probability of the conclusion (see Adams, 1975; Coletti & Scozzafava, 2002; Gilio, 2002).[17]

The most widely accepted criterion of probabilistic validity in the new paradigm, and also the most debated in philosophy, is Adams' (1998) *p-validity* which is defined by comparing the uncertainty of the premises and the uncertainty of the conclusion. Uncertainty is defined as the probability of falsity[18]: the uncertainty of an event A that has a probability P(A) is P(not-A) = 1 - P(A). *An argument is p-valid if and only if the uncertainty of its conclusion cannot exceed the sum of the uncertainties of its premises* (Adams, 1998, p. 131). This leads to partitioning the arguments into those that are p-valid and those that are not (the former obey the sum condition and the latter do not). Then, consider a p-valid argument: given the probabilities of its premises, the sum property allows the calculation of a limit in probability such that an individual who estimates the probability of the conclusion to be no lower than this limit can be regarded as possessing an implicit knowledge of the probability preservation of the argument. And reciprocally, a lower estimate indicates absence of such knowledge.[19]

Coherence. The second way of adopting Bayesian theory is to opt for de Finetti's (1937) notion of coherence. De Finetti (1937) provides an effective method to appraise the coherence of a probability evaluation by using coherence intervals determined by the probability of the premises and the static (respecting the axioms of additive probabilities) and dynamic (revising by Bayes's rule) criteria of coherence (Baratgin & Politzer, 2006)[20]. This approach is expounded in Coletti & Scozzafava (2002) and Hailperin (1996). It has been used by proponents of Probabilistic Mental Logic within the new paradigm in several investigations of deduction under uncertainty (Gilio & Over, 2012; Pfeifer, 2014; Pfeifer & Kleiter, 2009, 2010, 2011); see also Cruz, Baratgin, Over & Oaksford (2015), Evans, Thompson & Over (2015), and Singmann, Klauer, & Over (2014) for studies using both coherence and p-validity.

When applying the concept of coherence to inference schemas, it can be shown (Pfeifer & Kleiter, 2006) that again the arguments can be partitioned into two classes. For some arguments, called *probabilistically non-informative*, any assessment on the interval [0, 1] for the probability of the conclusion is compatible with the axioms of probability, whatever the probabilities of the premises. For the other arguments, called *probabilistically informative,* the interval is constrained by the probabilities of the premises. It has a lower and an upper limit [l, u], so that an individual presented with such arguments is deemed coherent if an only if his or her assessment falls inside the interval.

From a methodological point of view, it is clear that at the second level of knowledge also the new paradigm is far more demanding than the old one. For both approaches the probability estimates have to be fixed for the premises and measured for the conclusion, keeping in mind that for the latter the aim is to find the position of the participant's estimate with respect to a probability interval calculated by the experimenter. For coherence, the limits of the intervals can be calculated by applying the theorem of total probability in simple cases or by solving a system of linear equations (Coletti & Scozzafava, 2002). A very simple method applicable to the inference schemas commonly discussed in philosophical logic and in psychology is described in Politzer (2014).

Comparing p-validity and coherence. The main interest of the coherence approach stems from the fact that conceptually it justifies that the subjective degrees of belief attributed to the sentences can be regarded as additive probabilities (de Finetti, 1937). That is, that an individual's degree of belief in the conclusion lies inside the coherence interval means that he or she respects the laws of probability, which is the major question of interest on the investigators' agenda. In addition, this methodological approach offers a unified framework to study both deduction under uncertainty and probability judgment.

The method of coherence aims to study the participants’ probability estimate of the conclusion in relation with their estimate of the probability of the premises. This method allows to study, for each argument presented, whether participants produce a coherent evaluation. Then one can characterize the arguments according to the percentage of coherent evaluations observed across all the participants and differentiate the arguments for which the majority of the probability evaluations fall within the coherence interval (these arguments could be called "natural"), from those for which this is no the case.

The aim of p-validity differs. The question it aims to answer is whether an individual's degree of belief in the conclusion is high enough to respect a criterion of probability preservation that is used to characterize the argument as deductive. This comes in the wake of the old paradigm, whose main interest was the investigation of individuals' deductive abilities. In the new paradigm, it seems important to study whether reasoners judge that a high degree of belief in the premises warrants a high degree of belief in the conclusion, which is a property entailed by p-validy. Also, the p-validity measure may be used to differentiate, among probabilistically informative arguments, those which are deductive (p-valid) from those which are inductive (p-invalid, see Evans & Over, 2013).

However, the adoption of p-valdity raises several difficulties. One follows from the fact that the intervals determined by the two criteria generally do not coincide. Take for instance Modus Ponens: *A; if A then C; therefore C* and let P(A) = r and P(C/A) = q. The intervals are [max {r+q-1, 0}, 1] for p-validity and [rq; rq+1-r] for coherence; taking for example q = 0.5 and r = 0.8, the intervals are [0.3, 1] and [0.4, 0.6], respectively. Suppose two individuals give evaluations such as 0.35 and 0.7. They are both p-valid but incoherent, the former being too low and the latter too high. We may be satisfied that we have found two evaluations reflecting the deductive import of Modus Ponens, but both violate the laws of probability.

Another limitation of the p-validity measure is that as soon as the sum of the uncertainties of the premises is above some value (1 for two premises, 2 for three premises, etc.), no interval can be defined because by definition the uncertainty of the conclusion would be greater than 1 and so the probability negative.

Conclusion

A number of theorists have proposed Bayesianism as a norm of reference to investigate the psychology of reasoning. The ensuing new paradigm poses many new conceptual and methodological questions. The existence of a plurality of competing Bayesian formal models is one of the thorniest questions. The proposed solution is to distinguish the elementary level of knowledge (the level of the conditional event) from the meta-epistemic level (the level of the conditional probability). For the first level a comparative method that bears on the main trivalent systems has been presented. Our preliminary experimental results support the Finettian model. They indicate participants' strong preference for the truth tables defining the connectives of de Finetti's trivalent logic. In addition this model explains straightforwardly the classical "defective" truth table of the conditional.

For the second level, the proposed solution is to replace the traditional method based on validity by a method based on Finettian coherence. At this level our results confirm the hypothesis of the equivalence between assessing the probability of the conditional event and the conditional probabilty of the consequent conditioned on the antecedent, and we have shown that this obtains both in the situation of a conditional assertion and in the situation of a conditional bet. Because the second level concerns probabilistic evaluation, it is relevant for deductive inferences under uncertainty. Our ongoing research focuses on lay people's confidence in the conclusion of simple inference schemas using verbal probabilities, which has not been studied so far. Our observations (Politzer & Baratgin, submitted) confirm, and extend to other schemas, the results reported by Probabilistic Mental Logic investigators[21].

In this paper we have considered the major change brought about by the new paradigm, which primarily consists in taking uncertainty into account. There are, however, other key features of the new paradigm that may affect the methodology. One, reasoning cannot be reduced to a neutral mechanism independent of the individual's motivation, preferences and objectives (Bonnefon, 2009; Bonnefon & Sloman, 2013). Two, reasoning depends on the context: one reasons to and from an interpretation (Stenning & van Lalmbalgen, 2008). Three it depends on the individual's knowledge base and cognitive abilities (Stanovitch 2011).

Consequently it is important to pay heed to individual differences in the experimental studies. Individual subjective responses (judgments, and decisions) may be different but nevertheless coherent because based on different states of knowledge. Different responses may also reflect different mental processes (Bonnefon, 2013). The various factors at the origin of the subjectivity of beliefs such as cultural background should also be taken into account. The objectives of the new paradigm meet the foundation of the Bayesian subjective theory, and more precisely, in an explicit way, the Finettian theory. This convergence suggests that the new paradigm could be the solution to de Finetti's "true subjective probability problem" which is to understand "the ways in which probability is assessed and used by educated people, and the way in which such abilities can be improved" (de Finetti, 1974, p. 15).

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[1] The term “normative models” designates formal models that define a norm of behavioral rationality for reasoning, judgment and decision making. These models most of the time pertain to the fields of probability theory, economics, philosophy or artificial intelligence.

[2] The philosophical, linguistic, and also psychological literatures offer a variety of classifications of sentences of the type "if A, then C". One major distinction consists of separating subjunctive conditionals, that is, contrary to fact conditionals, from the indicative conditionals. We will be concerned only with the latter.

[3] This is the case for instance of the inference from the consequent to the conditional: *C, therefore if A then C.* (Bonnefon et Politzer, 2011; Hailperin, 1996; Pfeifer, 2014; Suppes, 1966).

[4] De Finetti keeps on warning against general formulations that often are at the source of misunderstanding. This is why in what follows we will refer to "de Finetti's model" to underscore the originality of his theory and specify it with regard to "Bayesian model", and even to "subjective Bayesian model".

[5] The expression "radical" to qualify de Finetti's point of view was used by Jeffrey (1993) and is now used by experts in de Finetti's work (see in particular Galavotti, 2008). Note that de Finetti himself was the first to regard and call his views as deeply "radical" (de Finetti, 1969, p. 6 and 8).

[6] The use of the expression "supposition" is motivated and has a long history. Bayes used it to define conditional probability, and Laplace when justifying Bayes' rule (see Edgington, 1995, p. 262 for the citations). It is for similar reasons that Hailperin (1996) called his theory "suppositional logic".

[7] Notice that at the end of the quote Ramsey (1929/1990) explicitly referred to a third truth value for the conditional, viz. "void" when the antecedent is false.

[8] This is the case, for instance, of Łukasiewicz's, Bochvar's, and Kleene's logics.

[9] Kleene's tri-valued logic coincides with Fi, except for the absence of the conditional event and for the fact that U explicitly stands for "indeterminate".

[10] This validity is often defined when U is interpreted as a value that stands for inconsistency (see for example Avron, 1991). This is the case of paraconsistent and relevant logics.

[11] Milne (2012) uses the same validity without specifying the trivalent system chosen.

[12] Using the author’s terminology, an event is U-valid if, in its truth table restricted to T and F, no value is equal to F and at least one value is equal to T. For instance, the conditional event A/A is U-valid because if A is T, A/A is T; and if A is F, A/A is U.

[13] With a system similar to Fi, Rescher (1969) showed that the definitions V_T and V_{NF} may lead to inconsistency when a value U or F is antidesignated.

[14] This is the case, for instance, of systems that take a different definition for conjunction such as quasi-conjunction where the composition of T and U yields T (Dubois & Prade, 1994, 1995; Milne, 1997). The desiderata mentioned above can be satisfied with a trivalent system other than de Finetti's, and a definition of validity other than V_U. This is the case, for example, of the trivalent system SAC (see Calabrese, 2002) using V_{TnF}.

[15] For a similar result, see DeRose & Grandy (1999, p. 166, note 9).

[16] Fugard, Pfeifer, Mayerhofer & Kleiter (2011) have observed a shift in interpretation towards the conditional as the sequence of the items proceeds. The fact that in the classic task the rate of conjunctive interpretation diminishes across age (Barrouillet, Gauffroy & Lecas, 2008) suggests that it is related to cognitive load or to limited cognitive ability; indeed it has been found that the conjunctive interpretation is more frequent among those who score lower on general ability tests (Evans, Handley, Neilens & Over, 2007; Sevenants, Dieussaert, & Schaeken, 2011).

[17] Coletti & Scozzafava (2002) and Gilio (2002) have the same interpretation of de Finetti's conditional event. However, they do not use the same definition of conjunction or the same value for the threshold.

[18] The expression "uncertainty" for the probability of falshood is somewhat ambiguous. Among various formalizations of non-additive measures of degrees of belief (which can be regarded as generalizations of additive probability) there exist several expressions for the degree of disbelief in an hypothesis A. Bel (not-A) is called degree of "incredulity", "doubt", "potential surprise" (for a review, see Haenni, 2009). The expression "uncertainty" is sometimes used (see Jøsang, 2001) for 1 - Bel(A) + Bel (not-A) which is null in the Bayesian framework. Note also that the definition of a valid inference in a non-additive framework amounts to Adams's p-validity: A entails C if Bel(A) is smaller than, or equal to, Bel(C).

[19] There is no strict coincidence between the inferential validity of the first level and the probabilistic validity of the second level. For instance, for SAC with V_{TnF} and V_U, transitivity is valid at the first level but not at the second level. (For other differences, see Milne, 2012).

[20] This method of calculation of an interval has a very old origin (for a review, see Hailperin, 1996).

[21] Although the Mental Probability Logic approach is methodologically close to what we are advocating here, we have some reservation with respect to the status of the coherence intervals. It seems doubtful to us that human beings possess the computational ability to assess or determine the intervals, which we view as a metacognitive ability. Rather than being part of a competence model, we view the coherence intervals as an effective tool --designed by the experimenter or the theorist-- to examine individuals' coherence, that is, whether their evaluation belongs to the interval. This is the way they were in fact used by de Finetti. Moreover, the mental representation of probability as an interval is a case of imprecise probability incompatible with the Finettian view (see de Finetti & Savage, 1962).