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Is the human mind definitely not Bayesian? A review of the various arguments

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Abstract

The various experimental studies conducted by psychologists on probabilistic reasoning tend to conclude that the "human mind is not Bayesian". However this conclusion appears debatable when one tries to shed light on the notion of Bayesian coherence. The main arguments of the literature illustrating the failure of human behaviour to adhere to the Bayesian model are analyzed. Conversely, the ineffectiveness of the studies themselves to fully comprehend what Bayesianism implies is extensively exposed.

Key words: probabilistic reasoning, Bayesian theory.

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Since the first study of Rouanet (1961) and following the famous article of Edwards, Lindman and Savage (1963), the Bayesian probabilistic model became a normative reference usually used by psychologists when conducting work on human probabilistic revision. The major part of psychological studies generally tends to show that human beings do not form their degrees of belief in a "Bayesian" way (see for example Piatelli-Palmerini, 1993).

The purpose of this paper is to analyze the ground on which the arguments supportive of such a hypothesis are based. Our aim is not to conduct a thorough review of the literature testing the experimental robustness of such arguments (see, for example, Baratgin, 1999) but rather to discuss the underlying idea that the "Human mind is definitely not Bayesian". First of all, we shall present the most often quoted arguments of the literature. Second, we will define the Bayesian model. Third, we will draw objective conclusions from the results presented in the psychology literature that adopt a Bayesian viewpoint.

MAN IS NOT BAYESIAN

Ten main arguments of the literature can be summed up by the same conclusion that "Human beings are not Bayesian". Subjects' predictions indeed deviate significantly from what experimenters compute using the theory of probability and Bayes' rule[1].

Conservatism

Participants in psychological studies are most likely to perform "revisions" that prove less optimal than those calculated by experimenters using Bayes' rule.

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1. Given two alternative hypotheses H and $\neg$H, one event D such that $D \neq \varnothing$, let P(H) be the "prior probability" of H, P(H|D) be the "posterior probability", P(D|H) and P(D|$\neg$H) be the "likelihoods" and finally P(D) be the "probability of the data" D. The standard form of rule's Bayes gives

$$\frac{P(H)P(D|H)}{P(H)} \text{ with } P(D)\neq 0.$$

This result labelled "conservatism" as initially introduced by Phillips and Edwards (1966) kept researchers busy for a decade (Edwards, 1968; Peterson & Beach, 1967; Phillips & Edwards, 1966; Slovic & Lichtenstein, 1971; see Baratgin, 1999, for a review). "Conservatism" illustrates the tendency of subjects to revise their initial judgements to a lesser extent than what Bayes' rule would require if they were to adhere to it. The posterior degrees of belief that subjects express remain closer to the corresponding prior probabilities than what the *posterior* probabilities computed according to *Bayes'* rule do. Nevertheless the directions of the revisions of the observed and Bayesian predictions are the same.

The experimental paradigms used are mainly based on the famous "Urn problem":

> "Two urns are filled with a large number of poker chips. The first urn contains 70% red chips and 30% blue. The second contains 70% blue chips and 30% red. The experimenter flips a fair coin to select one of the two urns, so the prior probability for each urn is .50. He then draws a succession of chips from the selected urn. Suppose that the sample contains eight reds and four blue chips. What is your revised probability that the selected urn is the predominantly red one?" (Peterson & Beach, 1967).

The "true result" for the experimenter when he/she applies Bayes' rule is a probability that lies around 1. This stems from the fact that *prior* probabilities are $P_{Experimenter}(H) = P_{Experimenter}(\neg H) = 0.5$. In addition, according to the binomial law, the likelihoods yield $P_{Experimenter}(D|H) = 0.231$ and $P_{Experimenter}(D|\neg H) = 0.008$, that is to say, $P_{Experimenter}(H|D) = 0.967$. Yet a sizeable majority of subjects gives the following answer $P_{Subject}(H|D) \approx 0.75$; which is significantly below what the experimenter gets using Bayes' rule.

Base rates neglect

A base rate is a numerical reference that describes the distribution of a character in a given population. It is often introduced in the form of a percentage or a frequency. Experimenters consider such pieces of information as the *a priori* degrees of belief that participants should adopt. Many studies however show that subjects tend to neglect base rates

(Kahneman & Tversky, 1972a, 1973, see Baratgin, 1999; Koehler, 1996, for a review).

The paradigm of medical test provides an example of this result:

> "1% of women at age forty who participate in routine screening have breast cancer. 80% of women with breast cancer will get positive mammographies. 9.6% of women without breast cancer will also get positive mammographies. A woman in this age group had a positive mammog-raphy in a routine screening. What is the probability that she actually has breast cancer? __ %" (Eddy, 1982).

The bulk of subjects gives the 80% response that illustrates the percentage of women with breast cancer that will get positive mammographies and thus seems to neglect the piece of information that 1% of women at age forty who participate in routine screening have breast cancer. On the contrary, the experimenter thinks the correct answer is 7.8%:

$$P_{Experimenter}(\text{breast cancer} \mid \text{posistive mammographies}) = \frac{0.01\times0.8}{0.01\times0.8+0.99\times0.96}.$$

Conjunction fallacy

According to the theory of probability when two events A and B are independent, the probability of the conjunctive event $A\cap B$ is the product of the two probabilities of events A and B. As a consequence, $P(A\cap B) \leq P(A)$, $P(A\cap B) \leq P(B)$. Several studies show that subjects tend not to repect such relations. " Linda's problem " originally presented by Tversky and Kahneman (1982) is the paradigm the more widely used to illustrate this phenomenon. Participants are given the following description:

> (E) "Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations."

They are then asked to rank various statements in the decreasing order of "the probability" they attach to them. Statements include these two:

> "(B)Linda is a bank teller.
>
> ($B\cap F$) Linda is a bank teller and is active in the feminist movement".

A great majority of subjects in the Tversky and Kahneman (1982) study ranks ($B\cap F$) as more probable than B (see Herwitg & Gigerenzer, 1999,

for a review) despite what can be expected from a strictly probabilistic point of view.

Insensitivity to sample size

The fallacy scrutinized here concerns the statistical representation of a sample vis-à-vis its parent population. More specifically, the degree of confidence one may hold in the ability of a sample to represent its parent population. According to the statistical theory, a sample is said to be representative of its parent population if it presents a sufficient range – quantitatively and qualitatively – of the characters under review of the parent population. The larger a sample, the more likely it is to be representative of its parent population. Yet subjects do not seem to take into account the size of samples when making judgements. This behaviour is called the "insensitivity to sample size" (Kahneman & Tversky, 1972b; see Sedlmeier & Gigerenzer, 1997, for a review). The more widely used experimental paradigm is the "Maternities problem":

> "A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50 percent of all babies are boys. However, the exact percentage varies from day to day. Sometimes it may be higher than 50 percent, sometimes lower. For a period of 1 year, each hospital recorded the days on which more than 60 percent of the babies born were boys. Which hospital do you think recorded more such days?" (Kahneman & Tversky, 1972b).

The majority of subjects believes that both maternities have the same chance of giving birth to boys. Only a thin minority chooses the second maternity, which is the correct answer in theory as the larger the sample the more likely it is to represent the characteristics of its parent population and, more specifically, the 50% proportion of male newborn.

Law of small numbers

Similarly to the "insensitivity to sample size", we find what Kahneman and Tversky (1972b) derogatorily call the "law of small numbers" as opposed to the "law of large numbers", which stipulates that the more

numerous the replications of an event the closer statistical estimates get to the initial parameters. Yet subjects tend to expect to find the same features in a short randomly drawn series of events as the ones they would get in an infinite series. The following paradigm typically exhibits this behaviour:

> "All families of six children in a city were surveyed. In 72 families the exact order of births of boys and girls was G-B-G-B-B-G.
> What is your estimate of the number of families surveyed in which the exact order of births was B-G-B-B-B-B?"
> (Kahneman & Tversky, 1972b).

A large majority of the participants who took part in the Kahneman and Tversky (1972b) experiment believes that the G-B-G-B-B-G series is more likely than the B-G-B-B-B-B series. Subjects rather retain the G-B-G-B-B-G series because it seems more chaotic and hereby representative of the randomness of gender. In the experimenter's eyes, the correct answer is however 72.

Gamblers' fallacy

The gamblers' fallacy consists in a special case of the "law of small numbers" that applies more specifically to series of random binary events (urn with balls of two colours, flip of a coin, light that flashes either to the left or to the right, ...). In an environment linked with gamble, man expects that a "success" outcome follows a "fail" one and vice versa. He believes in an equilibrium reversion denying the hypothesis that draws are independent. This phenomenon initially outlined by Laplace (1825, 1986), was later experimentally exhibited in various studies (see, for example, the work of Cohen and Hansel, 1955, 1957). This behaviour can also be evidenced in other contexts than games of chance, as did Kahneman and Tversky in several papers. Concerning a continuous character, people will expect the mean of the sample to be close to the mean of the parent population as this is the case in the following problem:

> "The mean IQ of the population of eighth graders in a city is known to be 100. You have selected a random sample of 50 children for a study of educational achievement. The first child

tested has an IQ of 150. What do you expect the mean IQ to be for the whole sample?" (Tversky & Kahneman, 1971).

The average IQ as calculated by experimenters is 101,

$$\left(\frac{(49 \times 100 + 150)}{50}\right).$$

Yet most subjects answer 100. According to Tversky and Kahneman (1971), participants are prone to the "gamblers' fallacy" because they believe that the IQ of the 49 other children will help and restore the mean of 100.

Anchoring and adjustment

When assessing probabilities human beings tend to base themselves on a particular piece of information, which they use as an anchor even though this information may not be relevant to the prediction under review. They will then proceed by making adjustments that are most of insufficient magnitude from this anchor. Tversky and Kahneman (1974) asked subjects to state the number of African countries that belong to the United Nations. The paradigm is a three steps procedure for each participant:

i. The experimenter turns a "Fortune wheel " so as to draw a number between 1 and 100.
ii. The experimenter then tells the participant whether the proportion of African countries is higher or lower than the number indicated by the " Fortune wheel ".
iii. The participant is expected predict the percentage of African countries that sit on the United Nations committee.

Results exhibit a strong correlation between the number found in step (i) and the prediction given by participants in step (iii). For example, if in step (i) the number is 65, then the percentage predicted on average is 45% and, respectively, 25% if the number in the first place is 10.

According to Tversky and Kahneman (1974), subjects grant too much importance to the number drawn in step (i) even if it is the mere fruit of chance. This is the number that will provide the anchor on which participants will base their prediction.

Unpacking effect

Research work shows that providing several descriptions of an event prompts different predictions of its probability. Typically the degree of belief in a given hypothesis rises when this hypothesis is introduced as the sum of other hypotheses (Fischhoff, Slovic, & Lichtenstein, 1978, see Tversky & Koehler, 1994 for review). For example, in Tversky and Koehler (1994), a first group of subjects predict the likelihood of dying from "unnatural causes" to be 32% while a second group of subjects believe that the likelihood of dying from an "accident", "homicide", and "other unnatural causes" are 32%, 10%, and 11%, respectively. When summed up the likelihood of dying from "unnatural causes" reaches 53% in the second group, which is significantly higher than the 32% found in the first place (see also Ayton, 1997; Koehler, Brenner, & Tversky, 1997). This phenomenon, called the unpacking effect by Tversky and Koehler (1994), shows that predictions of probabilities can be "subadditive" (the probability of an explicit hypothesis is less or equal to the sum of the probability of the components).

Non compliance with the constraint of complementarity

Several studies show that subjects tend to give answers that violate complementarity, a property directly derived from the additivity of probability ($P(H)+P(\neg H)=1$). Let us consider a revision task where subjects are expected to estimate the probability of the guilt of three suspects. Participants know that the culprit is bound to be one of the three suspects. Yet the individual triplets' estimates do not generally add up to one (Baratgin, 1999; Robinson & Hastie, 1985; Schum & Martin, 1982; Teigen, 1983; Van Wallendael, 1989; Van Wallendael & Hastie, 1990). This phenomenon appears quite common in the case of the prediction of two complementary alternatives (see for example Tversky & Koehler, 1994). It can also be observed when more than two alternatives are possible. In any case, one can either get a superadditivity (the sum of the probabilities of the different alternatives is less than one) or a subadditivity (the sum of the probabilities of the different alternatives is more than one) (see for example Ayton, 1997; Cohen, Dearnaley, & Hansel, 1956; Macchi, Osherson, & Krantz, 1999; Villejoubert & Mandel, 2002; Windschitl, 2000).

Resuscitated hypotheses

In successive revisions, subjects tend to re-allocate a non-null probability to hypotheses they first held to be impossible (Curley & Golden, 1994; Robinson & Hastie, 1985; Van Wallendael, 1989; Van Wallendael & Hastie, 1990). Let us pursue the above paradigm and suppose that a participant initially gave a probability of guilt of a given suspect equal to zero. When further to a new message this participant re-estimates the probability of guilt of the same suspect as being strictly positive, we say s/he resuscitates this hypothesis. This result seems to contradict a feature of Bayes' rule. If one holds the view that a given hypothesis is impossible ($P(H)=0$), one will always attach a null probability to this hypothesis when revising on the basis of Bayes' rule, whatever the new message is (Zadeh, 1979; Dubois & Prade, 1985).

These ten arguments are generally presented and discussed in the literature as fallacies. Meanwhile, no clear definition of Bayesianism is to be found in this literature (Baratgin, 1999). The term of "Bayesian" is almost always confined to the simple use of Bayes' rule. This vagueness in the interpretation of the Bayesian model can only be harmful to experimental work. Our purpose is now to define Bayesianism and to confront the ten arguments with this definition.

BAYESIANISM

In the defence of psychologists, it seems extremely difficult to clarify in a unique way Bayesianism (Baratgin, 1999). First of all, the heterogeneity of the various interpretations of probability that underlie Bayesianism, is at the origin of the numerous forms of the Bayesian theory (Good, 1971, 1975; Jeffrey, 1983; Kaye, 1988). Second, misunderstanding is for a large part enhanced by the confusion which has become widespread between Bayesianism and all the theories that use Bayes' rule or one of its multiple by-products, that is to say, a confusion between the "Bayesian standpoint" and "Bayesian techniques" (de Finetti, 1974).

Bayesian standpoint

It is most important to clarify the "Bayesian standpoint" before conducting experimental work. It refers to a concept of internal coherence of human judgement. One can give a general definition of it according to three characteristics. A general epistemic criterion and two criteria defining a notion of coherence which degrees of belief are expected to comply with (Baratgin, 1999; Christensen, 1992; Good, 1973/1976; Kaplan, 1989; Seidenfeld, 1979). The double-faced coherence rests, on the one hand on a "static hypothesis", and on the other hand on a "dynamic hypothesis" (Hacking, 1967).

Epistemic criterion. The theory of probability provides a model of degrees of belief. Everybody is supposed to be able to attribute a personal distribution of precise probability to any event. This should be so even in a situation of total uncertainty or ignorance. According to this idea, probability is the measure of the degree of belief depending on the (general) state of knowledge of an individual, at a given moment. Consequently, such a measure is completely subjective.

Criterion of static coherence. Degrees of belief have to obey the axioms of the theory of additive probability (Kolmogoroff's axioms). They must comply with the two following "rules of convexities":

Let $\varnothing$ be the impossible event and Ω the set of possible results.
Let H and G be two events or hypotheses and D be a piece of information such as a description:
(i) $P(\varnothing) = 0$ and $P(\Omega) = 1$;
(ii) H is said to be "significant" if, and only if, $0 < P(H) < 1$;
(iii) degrees of belief must comply with the following additivity rule,
If $H \cap G = (\varnothing)$, then $P(H \cup G) = P(H) + P(G)$
as well as with the corollary constraint of complementarity,
$P(H) + P(\neg H) = 1$.

The conditional probabilities must satisfy the rule of compound probabilities:

(iv) $P(H \cap D) = P(H)P(D \mid H)$;

or its corollary named Bayes' rule:

$$\text{(v) } P(H \mid D) = \frac{P(H)P((D \mid H)}{P(D)} \text{ with } P(D) \neq 0.$$

Dynamic coherence criterion. Revision of an individual's degree of belief given by new evidence D is derived from Bayes' rule. This revision can be conducted in two different ways.

First, one can imagine D actually occurred at a time t_1 (with $t_0 < t_1$) and estimate the posterior probability $P(H|D)$ at a time t_0, although one has no knowledge at t_0 whether D will occur or not.

Second, one may revise ones' degrees of belief on learning the realization of D. One would then revise P(H) at a time t_2 (with $t_0 < t_2$) when one is informed that D occurred at a time t_2. Let this probability be $P_D(H)$.

The dynamic coherence criterion also called the "*(Bayesian) conditioning principle*" (Hacking, 1975; Seidenfeld, 1979) indicates that the revised probability when the announcement of D is made at time t_2 is equal to the probability of H conditioned on the (imagined or assumed) evidence D at a moment t_0; that is, $P(H|D)$ yielded by Bayes' rule.

(vi) $P_D(H) = P(HD)$.

Two sorts of Bayesianism

Traditionally two sorts of "Bayesianism" are distinguished: A "subjective Bayesianism" and a "logical Bayesianism" (de Finetti, 1959; Hacking, 1975; Seidenfeld, 1979). Both types of Bayesianism represent "degrees of belief" by probabilities conditional to a state of knowledge and respect naturally the two criteria of static and dynamic coherence.

Subjective Bayesianism. For subjectivists, probability depends on the general state of mind, which the individual forming this belief is in. Two agents having the same state of knowledge are thus authorized to emit two different predictions of probability as long as their estimates are coherent (in the static and dynamic sense). In other words, there is nothing such as a unique function of probability for a given state of knowledge. Allowing for states of mind bears further consequences as far as the dynamic process of revision of degrees of belief is concerned. Indeed, it implies that some differences can exist in the way two people learn the same message. There is subjectivity in the learning of knowledge (Baratgin, 1999; Shafer, 1985). This theory was thoroughly investigated by Ramsey (1926, 1931) and independently in a slightly different light by de Finetti (1931a, 1931b, 1936, 1937).

Logical Bayesianism. "Logical Bayesians" (for example, Cox, 1946; Jaynes, 1994; Jeffreys, 1931; Keynes, 1921 – the founder) formulate degrees of belief that do not depend on the state of mind of subjects. In particular, according to "logical Bayesians" two individuals having the same state of knowledge make the same judgement of probability. As a consequence, in the context of the dynamic process of probabilistic revision, two individuals who have the same state of knowledge and to whom one passes on the same message are supposed to have the same new knowledge.

These two types of Bayesianism feature the same prescriptive aspect. The theory should be a guide for human reasoning (see, for example, de Finetti, 1965; Jaynes, 1998). However, only subjectivists sometimes confer to Bayesianism an ability to describe reality (see de Finetti, 1937, 1955, 1961). For this reason and because psychologists explicitly refer to "subjective Bayesianism" (for example: Edwards, Lindman, & Savage, 1963; Kahneman, Slovic, & Tversky, 1982), it is the subjective interpretation of Bayesianism that will be considered from now on.

BAYESIAN STANDPOINT AND THE TEN ARGUMENTS OF THE LITERATURE

According to the general criteria of Bayesianism, assessing the Bayesian character of an individual amounts above all to analyzing his/her degrees of belief. However this is not so much a question of looking at the quality of his/her answers with respect to an objective result. One can very well be "coherent" from a Bayesian standpoint and give an evaluation very remote either from the final outcome of the event under review or from the evaluation supplied by an expert. Similarly even though they were given the same information, two people can emit different degrees of belief and be both coherent from a Bayesian point of view as long as the three criteria are adhered to.

The three Bayesian criteria we identified constitute the new light which we propose to shed on the ten main arguments of the psychological literature that supposedly support the idea that man is not "Bayesian". In particular, we will confront the idea of Bayesianism that the arguments presuppose with the criteria we defined. We will see that six arguments have to be reconsidered in the light of the epistemic cri-

terion, three in the light of the static coherence. The last argument will be significantly waived down in front of the dynamic coherence.

Epistemic criterion

Six out of the ten arguments described above ("conservatism", "base rates neglect", "insensitivity to sample size", "law of small number", "gamblers' fallacy", and "anchoring and adjustment") do not actually refer to an epistemic representation of probability. These results pertain to studies using a methodology of calibrating with respect to a result calculated by the experimenter using Bayes' rule and probabilistic knowledge. In these six arguments, the experimenter proceeds as if a unique "objective" probability was the only possible answer to the requested prediction. Whereas as mentioned above, Bayesians consider that probabilities are conditional to a dual state of knowledge and mind. Following this idea, there is nothing as a probability on its own does not exist independent from the assessments of subjects (de Finetti, 1970, 1976). One cannot talk about an "objective" or "correct" or "true" probability. As a consequence, these six arguments do not actually appear valid in testing the hypothesis of man being Bayesian.

Conservatism and base rates neglect. In the paradigms of pp. 654-655, the experimenter finds base rates and diagnostic information relevant. Now in all probabilistic assessment, subjects use the information they judge relevant for themselves. Therefore base rates that the experimenter retains as prior probabilities, are not necessarily considered as such by participants (Baratgin, 1999, 2002; Levi, 1981, 1983; Logue, 1995; Niiniluoto, 1981; Phillips, 1970). On the contrary, the experimental paradigms used make it difficult for subjects to analyze base rates as prior probabilities. For example, in the medical test, no detail is provided on the reliability of base rates. We do not know if the "women in this age group" come from the same group as the one on which base rates have been computed. The same applies to likelihoods, to the probability of the message, and as a consequence, to posterior probabilities subjects are expected to provide. In the urn problem too, likelihoods used by subjects can be different from the ones calculated by the experimenter on the basis of the binomial law. Participants in these experiments can be perfectly "Bayesian" and still form predictions that are

different from what the experimenter expects because they use different information. For instance, Baratgin (2002) shows that participants use different prior probabilities than the base rates given by the experimenter. An appropriate methodology would be first of all to try and establish the prior probabilities and the likelihoods that participants actually use. This could be achieved by asking subjects for a primary assessment before giving them any diagnostic information. As a matter of fact, in the literature several studies exhibit better participant performance when the experimenter takes greater care in the way s/he delivers the information:

Firstly some researchers condemned the fact that experimental paradigms for pragmatic reasons induce subjects in the wrong direction. As for the base rates neglect paradigm, thanks to pragmatic tools used to reword statements subjects' performance has been significantly improved (for a review, refer to Baratgin, 1999; Hilton, 1995; Hilton & Slugoski, 2000; Politzer & Macchi, 1999). If one manages to make the key parameters (base rates, likelihood) that the experimenter uses in his/her calculus, relevant to subjects then participants' answers get closer to the ones calculated using Bayes' rule.

Second human beings seem to feel more comfortable when numeric references (and in particular base rates) are presented in terms of natural frequencies (see Hoffrage, Gigerenzer, Krauss, & Martignon, in press for review). For example, let us put the medical test paradigm (p. 656) in a frequency format:

> "10 out of every 1000 women at age forty who participate in routine screening have breast cancer. 8 of every 10 women with breast cancer will get positive mammographies. 95 out of 990 women without breast cancer will also get positive mammographies. A woman in this age group had a positive mammography in a routine screening. What is the probability that she actually has breast cancer? __%" (Gigerenzer & Hoffrage, 1995).

In this context subjects tend to give answers that get significantly closer to what the experimenter expects when applying Bayes' rule[2]. The frequency format presents information (such as base rates) as if naturally observed, rather than abstractly fixed by the experimenter (Hoffrage et al., in press).

More generally hinting at a probabilistic interpretation of problems prompts participants to use the same information as the one the experi-

menter uses. When information is presented in a visual form subjects' answers get closer to the Bayesian norm. The introduction of a pie chart to represent base rates improves their use among participants (Baratgin, 2002). More generally when one clarifies the nested-set relation of the paradigm to participants, their performance is enhanced. It is so, for example, when the implementation of an Euler circle diagram illustrates how the events relate to one another in the medical test paradigm (Sloman, Over, & Slovak, in press).

Finally the few studies that use an appropriate methodology to study Bayesian revision (i.e. taking into account the prior probabilities and the likelihoods actually considered by subjects) show that participants' predictions come close to the ones calculated thanks to Bayes' rule (Baratgin, 2002; Evans, Brook, & Pollard, 1985; Peterson, Ulehla, Milner, Bourne, & Stilson, 1965; Scholz, 1987; Schum & Martin, 1982; see Baratgin, 1999, for a review).

Anchoring and adjustment. The "anchoring and adjustment" argument does not appear as a real proof that man is not Bayesian. Human beings may indeed base their judgements on a piece of information that the experimenter does not find relevant for the prediction under review. Subjects' behaviour can easily be understood for they are indeed asked a prediction in a field they do not know at all (percentage of African countries sitting on the United Nations committee). The fact that the experimenter makes a Fortune wheel turn can lead subjects to think this action has to do with the prediction. Otherwise why would the experimenter give this information?

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2. The frequency format leads to a computing simplification of the breast cancer paradigm. Bayes' rule in this format yields:

$$P(\text{disease} \mid \text{symptom}) = \frac{\frac{8+2}{8+2+95+895} \times \frac{8}{8+2}}{\frac{8+95}{9+2+95+895}} = \frac{8}{8+95}.$$

As a consequence the posterior probability only depends on the 8 and 95 numbers that come from the diagnostic message. The frequency format does not seem to consist in a real proof that subjects use base rates since in this condition base rates cancel out from the computation (Kleiter, 1994).

Insensitivity to sample size, law of small number, and gamblers' fallacy. These three arguments refer to an environment of "chance" and to the view one holds of it. The experimenter knows about the "law of large numbers" but participants may have a different idea of "chance". Therefore, since according to Bayesians there is no true prediction of probabilities, subjects' degrees of belief may differ from the experimenter's ones. From a Bayesian point of view the two types of prediction can be equally coherent even though participants' estimates are based on a conception of chance that may be perceived as erroneous or even completely irrational to statisticians' eyes. These three arguments should be analyzed as a testimony for the lack of statistical culture among subjects (see Cohen, 1979, 1982) than as a proof that man is not Bayesian[3]. That was what de Finetti (1955) meant when he responded to similar critics:

> "Le ... problème ...des évaluations habituelles de la théorie des probabilités dans des questions où l'on peut appliquer les critères des cas "également probables" ou de l'évaluation "d'après une fréquence observée" ... n'a aucun rapport avec les fondements de la théorie subjective des probabilités. Les axiomes de cette théorie imposent seulement, par exemple, à un joueur à la roulette, d'attribuer à la sortie 0, 1, 2, ..., 36 dans un coup donné, des probabilités non négatives de somme égale à un, s'il les juge égales entre elles (1/37), ou inégales pour n'importe quelle raison (bien ou mal fondée d'après le jugement commun), cela ne touche pas aux axiomes. L'individu qui jugerait très probable, au prochain coup, la sortie du "13", par exemple, parce qu'il ne s'est pas présenté depuis longtemps, ou parce qu'un joueur qui a beaucoup de chance va le jouer, ou parce qu'il en a lui-même reçu la suggestion en rêve, nous pouvons le qualifier de fou en tant qu'il attribue de l'importance à des choses qui ne le méritent pas, mais nous ne pouvons pas le charger d'une faute par rapport aux axiomes."

Moreover one can still make "Bayesian" sense out of subjects' answers (Logue, 1995). Let us first consider the experimental paradigm of the "law of small numbers". In the situation of the series of births, if

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3. Several studies actually show that subjects possess some real intuitive statistical abilities (see for review Sedlmeier & Gigerenzer, 1997).

one feels confident in one's knowledge that the true long term expected proportion of boys is 1/2 and if one's degrees of belief is not modified after each birth, then one's degree of belief must be the long run expected frequency of 1/2. Therefore if one is presented with an asymmetrical series for which the frequency of a boy is far from 1/2, one will reasonably think that this series is unlikely. This is a classical result of de Finetti (1937) reformulated several times by the author (see, for example, de Finetti, 1970, 1972, 1974) as well as by Jeffrey, who surprisingly called it "de Finetti's law of small numbers" (Jeffrey, 1983). The same logic applies to participants' answers to the maternity hospitals and IQ problems.

Static coherence

The second criterion of static coherence commands us to study whether subjects respect probabilistic axioms. We will test in particular participants' compliance with the complementarity constraint. De Finetti (1955) explains how psychologists should proceed to study static coherence:

> "Le deuxième problème concerne la cohérence des individus, c'est-à-dire l'hypothèse qu'ils se conforment aux axiomes, en tant que question de fait. On y considère les cas où la discordance tient à l'incapacité d'effectuer des raisonnements ou des calculs, en les distinguant de ceux où il s'agit d'un désaccord véritable entre l'esprit de l'individu et les axiomes en discussion ... La discordance est manifeste si elle se produit dans des exemples très simples, quand elle ne peut résulter de difficultés de calcul ou de raisonnement. Elle est délibérée si l'individu persiste dans une évaluation qui ne respecte pas les axiomes même après que son attention a été attirée sur ce point".

In this respect, we will analyze three arguments that seem relevant to assess the Bayesian character of individuals; the "unpacking effect", the "non respect of complementarity", as well as the "conjunction fallacy".

Unpacking effect. Human beings attach greater degrees of belief to hypotheses that consist several disjunctions. In the experimenter's eye, participants in the experiment (p. 660) think that

$$P_{Experimenter}(\text{unnatural causes}) < P_{Experimenter}(\text{accident} \vee \text{homicide} \vee \text{other unnatural causes}),$$

and therefore violate the axiom of additivity of probabilities. This result is then categorized as a fallacy since experimenters consider the two probabilities equal. From a Bayesian standpoint, probabilities are conditional to the information given (as well as to subjects' knowledge). For the first group, message D is "unnatural causes" that participants can understand in various ways. However for the second group, an explanation of the "unnatural causes"(message D') is given. As a consequence one cannot expect predictions to be the same and it can be the case that

$$P_{Subject}(\text{unnatural causes} \mid D) < P_{Subject}(\text{accident} \vee \text{homicide} \vee \text{other unnatural causes} \mid D').$$

In some cases a "superadditivy" can even be found whereby the inverse relation holds in the participants' eyes (see for example Hadjichristidis, Stibel, Sloman, Over, & Stevenson, 1999).

In addition, even if we were in a situation where D and D' are perceived as equivalent by subjects, a methodological problem would still prevent us from concluding that participants violate the additivity axiom. The main studies dealing with "unpacking effect" (for instance Ayton, 1997; Fischhoff et al., 1978; Koehler et al., 1997; Tversky & Koehler, 1994), compare the mean predictions of two groups of subjects (between-subject analysis). The experimenter proceeds in fact as if there were only one possible answer for all participants. As mentioned above, this is not a Bayesian standpoint. A within-subjects analysis seems necessary in order to conclude that there is a possible violation of the additivity axiom.

Complementarity constraint. Numerous studies evidence a non-compliance with the complementarity constraint. Either using a between-subject analysis (for example Ayton, 1997; Fischhoff et al., 1978; Koehler et al., 1997; Macchi et al. 1999) or a within-subjects analysis (for example, Baratgin, 1999; Baratgin & Noveck, 2000; Robinson & Hastie, 1985; Sherman, McMullen, & Gavanski, 1992; Van Wallendael & Hastie, 1990; Villejoubert & Mandel, 2002; Windschitl, 2000). According to the point made above, only the latter studies prove relevant from a Bayesian standpoint. Now, is this violation of the complementarity constraint a robust phenomenon illustrating the incapacity of the human mind to adhere to the basic Bayesian constraint, or is it the result of a discrepancy between the experimenter and the subjects' degrees of

belief? In the literature, few authors propose an explanation for this non-compliance with the complementarity constraint. Essentially, three explanations can nevertheless be put forward.

First, the tendency of subjects not to comply with the axiom of additivity can be accounted for by their neglect of alternative hypotheses. On the one hand, human beings may conceive hypotheses individually instead of as a complementary set. Hence probabilities of these hypotheses can end up varying independently from the others (McKenzie, 1998; Teigen, 1983; Van Wallendael, 1989; Van Wallendael & Hastie, 1990; Windschitl, 2000). On the other hand, subjects can focus on only one hypothesis (Sanbonmatsu, Posavac, & Stasney, 1997; see Windschitl, 2002, for a review). As a consequence in the case of two complementary alternatives, the violation of the complementary constraint can be significantly diminished by asking participants to assess the probability of both hypotheses (Baratgin & Noveck, 2000).

Second, the violation of the constraint of complementarity in non-binary situations could be a direct consequence of the subadditivity induced by an "unpacking effect". The alternatives to a focal hypothesis tend to be left unpacked as implicit disjunction (Koehler et al., 1997; Tversky & Koehler, 1994; see Windschitl, 2000, for a review). Therefore following the explanation above, this result seems questionable from a Bayesian standpoint.

Third, neglect of the axiom of additivity can also stem from subjects' misunderstanding of conditional probabilities (Baratgin, 1999; Sherman, McMullen, & Gavanski, 1992; Villejoubert & Mandel, 2002). For instance, subjects may estimate the following probabilities $P(D|H)$ and $P(D|\neg H)$ instead of $P(H|D)$ and $P(\neg H|D)$ requested by the experimenter. In this case, it would no longer be a problem of violation of the constraint of complementarity. Consequently, further understanding of this confusion is required to conclude whether this apparent violation of static coherence by subjects corresponds to a real violation of the complementarity constraint.

Conjunction fallacy. The rule of compound probabilities (point (iv) of the static coherence), yields the following *conjunction rule*: $P(H \cap D) \leq P(H)$ and $P(H \cap D) \leq P(D)$. Linda's problem leads to the conclusion that participants violate this rule. However, the literature offers several explanations for this result.

Some authors claim that subjects' predictions are actually coherent with respect to the pragmatic context of the paradigm (Adler, 1984, 1991; Dulany & Hilton, 1991; Hertwig & Gigerenzer, 1999; Hilton, 1995; Politzer & Noveck, 1991). Politzer and Noveck (1991), for instance, find that half the participants actually compare $P(B \cap F)$ and $P(B \cap \neg F)$ when asked to compare $P(B \cap F)$ and $P(F)$. According to this argument, there would no longer be a violation of the conjunction rule. Some other researchers think that subjects do not understand the task they are given as an assessment of a probability but rather they see it is a task of confirmation (degrees of confirmation or support). In this case, too, it would no longer be a fallacy (see Hertwig & Chase, 1998; Sides, Osherson, Bonini, & Viale, in press).

Nonetheless, only few authors argue that subjects' predictions can be viewed as Bayesian in the original experiment. Hintikka (1999) provides us with a Bayesian understanding of Linda's problem that we summarize here. In a situation where two messages A and B come from two different sources, A and B can naturally be assessed with respect to two different distributions of prior probabilities P and P'. Therefore we can have the following results:

$P(A \mid E) < P(B \mid E)$,
$P'(A \mid E) < P'(B \mid E)$
and $P'(B \mid E) < P(A \mid E)$.

Hintikka analyzes Linda's problem in a similar way. In the original experiment, participants may have found it difficult to believe that the two propositions B and $(B \cap F)$ stemmed from the same source of information. They may actually consider that the two propositions B and $(B \cap F)$ come from different sources of information. Participants may be conceiving a scenario of two witnesses answering the question: What does Linda do now? A first witness would propose "B" while a second one would bring about "$B \cap F$". In the light of the E information given to subjects, the witness thought to have furnished the "$B \cap F$" proposition was perceived as being more reliable than the one who limited himself to the "B" proposition. In accordance with this, the former witness may sound more precise and hence more knowledgeable.

As Hintikka suggests, the above explanation is consistent with the fact that when Linda's problem is cast in a frequentist format, only a minority of subjects commits the "conjunction fallacy". The frequency format of Linda's problem reads as follows:

"In an opinion poll, the 200 women selected to participate have the following features in common: [description Linda 's].
How many of the 200 women are bank tellers? __ of 200."
(Fielder, 1988).

In this context no room is left for subjects to think of a two witnesses scenario. This explanation is also consistent with Sloman et al.'s (in press) result of where asking participants to assess the probabilities of "B" and "B∩F" (rating) rather than ordering them (ranking) as in the standard condition significantly improved performance. In the assessment situation, the two witnesses scenario seems unlikely.

Dynamic coherence

Finally, the criterion of dynamic coherence assumes that revision is conducted according to Bayes' rule. The tenth argument on "resuscitated hypotheses" can thus be implicitly analyzed as a violation of this criterion. One can evoke a violation only in the case where subjects are in the first place convinced of the impossibility of the event under review ("null" prediction). More detailed research should be conducted to analyze subjects' "null" degrees of belief before testing the "resuscitated hypotheses" argument. Do these degrees of belief represent a real impossibility, or are they just the result of rounding effects of predictions of very small probabilities (something around 10^{-3} for instance)? In the latter case, the "resuscitated hypotheses" argument would no longer hold.

Of the ten arguments from the psychological literature generally put forward to support the idea that man has a non-Bayesian mind, the neglect of the constraint of complementarity seems to be the most relevant. It clearly contradicts the criterion of static coherence. Nevertheless, some studies investigating this field have shown that if one prompts subjects to respect the constraint of complementarity, their revision appears in the meantime close to what yields Bayes' rule. For instance in a revision task, when participants are reminded that the sum of predictions of both hypotheses (focal and complementary) must be 1 (or 100%), subjects' performance improve (Baratgin, 1999, 2002; Baratgin & Noveck, 2000). Can we then conclude to a definite violation of

Bayesianism, if subjects improve their performance when cast in a probabilistic set up (de Finetti, 1955)?

CONCLUSION: HAS MAN A BAYESIAN MIND? AN OPEN QUESTION

We strove to analyze the arguments most frequently put forward in the psychological literature to assess whether the human mind is Bayesian. It seems yet premature to firmly conclude about the non-Bayesian character of human beings. More exactly, we did not find in the psychological literature substantial evidence to contradict the hypothesis that man may have a subjective Bayesian mind. Eight out of the ten discussed arguments are instances of an objective interpretation of probabilities on behalf of the experimenter. Therefore, participants' answers cannot be considered as biases from a Bayesian point of view. Nevertheless, we certainly acknowledge that the question actually remains open for lack of an adequate methodology based on the subjective Bayesian model that would enable researchers to address the question properly. Are violations of complementarity, and to a lesser extent the resuscitated hypothesis argument true violations of Bayesianism or can they be simply accounted for by pragmatic aspects?

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RÉSUMÉ

Les nombreuses études expérimentales sur le raisonnement probabiliste, menées par les psychologues, laissent à penser que "l'être humain n'est pas bayésien". Cette conclusion, cependant, est remise en cause

lorsque l'on affine la notion de cohérence bayésienne. Les principaux résultats de la littérature qui militent en faveur d'un comportement non bayésien sont analysés. Réciproquement, l'incapacité de ces études à prendre en compte les implications du modèle bayésien est étudiée en détail.

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