Measuring coherence in reasoning under uncertainty

Nicole Cruz. Birkbeck, University of London and EPHE

Reasoning under uncertainty: The probabilistic approach

- Most reasoning both in everyday life and in science parts from premises that are uncertain: held only with degrees of belief (Elqayam & Over, 2013; Oaksford & Chater, 2010).
Reasoning under uncertainty: The probabilistic approach

• Most reasoning both in everyday life and in science parts from premises that are uncertain: held only with degrees of belief (Elqayam & Over, 2013; Oaksford & Chater, 2010).

• The degree of belief in a conditional statement *If* $p$ *then* $q$ is a function of the conditional probability of $q$ given $p$ (Adams, 1998; Edgington, 1995):

  → The Equation: $P(\text{if } p \text{ then } q) = P(q | p)$
Reasoning under uncertainty: The probabilistic approach

- Most reasoning both in everyday life and in science parts from premises that are uncertain: held only with degrees of belief (Elqayam & Over, 2013; Oaksford & Chater, 2010).

- The degree of belief in a conditional statement \( \text{If } p \text{ then } q \) is a function of the conditional probability of \( q \) given \( p \) (Adams, 1998; Edgington, 1995):

\[
P(\text{If } p \text{ then } q) = P(q | p)
\]
Reasoning under uncertainty: The probabilistic approach

- Most reasoning both in everyday life and in science parts from premises that are uncertain: held only with degrees of belief (Elqayam & Over, 2013; Oaksford & Chater, 2010).

- The degree of belief in a conditional statement *If p then q* is a function of the conditional probability of *q* given *p* (Adams, 1998; Edgington, 1995):

  \[ P(\text{If } p \text{ then } q) = P(q \mid p) \]

- People compute this probability by performing a *Ramsey test* (Evans & Over, 2004; Ramsey, 1929/1994; Stalnaker, 1968).
The previous binary approach

- Could represent only certain truth and falsity, with no degrees of belief in between.
The previous binary approach

• Could represent only certain truth and falsity, with no degrees of belief in between.
• Could represent only unconditional, factual propositions. The truth or falsity of a composite proposition \( \text{if } p \text{ then } q \), was a function of the truth or falsity of its constituent propositions \( p \), and \( q \) (Johnson-Laird & Byrne, 1991, 2002)

<table>
<thead>
<tr>
<th>( \text{if } p \text{ then } q )</th>
<th>( p, q )</th>
<th>True</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p, \neg q )</td>
<td>False</td>
<td></td>
</tr>
<tr>
<td>( \neg p, q )</td>
<td>True</td>
<td></td>
</tr>
<tr>
<td>( \neg p, \neg q )</td>
<td>True</td>
<td></td>
</tr>
</tbody>
</table>
The previous binary approach

- Could represent only certain truth and falsity, with no degrees of belief in between.
- Could represent only unconditional, factual propositions. The truth or falsity of a composite proposition \( \text{if } p \text{ then } q \), was a function of the truth or falsity of its constituent propositions \( p \), and \( q \) (Johnson-Laird & Byrne, 1991, 2002)

<table>
<thead>
<tr>
<th>( p, q )</th>
<th>( \text{if } p \text{ then } q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>False</td>
<td>False</td>
</tr>
<tr>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>True</td>
<td>True</td>
</tr>
</tbody>
</table>
The normative system question

• When investigating reasoning from uncertain premises, we require criteria for the correctness of an inference that take account of uncertainty.
Binary consistency

• In the binary approach to reasoning, a central criterion for the correctness of an inference was given by whether the statements involved in the inference were consistent or not: The absence of a contradiction.
Binary validity

• A second central criterion for the correctness of an inference in the binary approach was validity.
Binary validity

• A second central criterion for the correctness of an inference in the binary approach was validity.
• An inference is valid iff it would be inconsistent to assume that the premises are true but the conclusion false.
Binary validity

• A second central criterion for the correctness of an inference in the binary approach was validity.

• An inference is valid iff it would be inconsistent to assume that the premises are true but the conclusion false.

→ binary validity is truth preserving.
Probabilistic criteria for inference correctness

• A central development in the probabilistic approach was the generalisation binary consistency to coherence, and the generalisation of binary validity to probabilistic validity, \textit{p-validity}. 
Coherence

• An inference is coherent when it complies with the axioms of probability theory (de Finetti, 1936).

\[
P(A) = 0.6 \\
\Rightarrow P(\text{not } A) = 0.4
\]
Justifications for coherence: Dutch Books

• If a person acts in an incoherent way, then a Dutch book can be made against her: A series of bets that are guaranteed to lead to a net loss for her, independently of the outcome of the bets (de Finetti, 1936; Ramsey, 1926/1994).
Justifications for coherence: Water tank analogy

• Also, if a person acts incoherently, this is as if she would pour liquid into compartments of a tank in a way that violates physical laws (Politzer, 2014).
Intervals for coherence

• Given the probabilities of the premises, the conclusion is coherent if it falls within a certain probability interval.
• If the premises are very informative for the conclusion, the interval can reduce to a point value.
• If the premises are non-informative for the conclusion, the interval extends to the whole probability range.
Coherence: Example

• Linda is a feminist and a bank teller

• Therefore, Linda is a bank teller.

Tvertsky & Kahneman (1983)
Coherence: Example

• Linda is a feminist and a bank teller

• Therefore, Linda is a bank teller.

\[ P(B) \in [P(F \& B), 1] \]

Tversky & Kahneman (1983)
P-validity

• Let the uncertainty of a statement equal 1 minus its probability: \( U(A) = 1 - P(A) \).

• Then an inference is \( p \)-valid iff there are no coherent assignments of probabilities to the premises and conclusion in which the uncertainty of the conclusion is greater than the sum of the uncertainties of the premises (Adams, 1998).
P-validity: Example

• Linda is a feminist and a bank teller ____%
• Therefore, Linda is a bank teller. ____%

\[ P(B) \in [P(F \& B), 1] \]
Coherence and p-validity

• The threshold for p-validity corresponds to the lower bound of the interval for coherence.
• P-validity has no upper bound.
Coherence and p-validity

• The threshold for p-validity corresponds to the lower bound of the interval for coherence.
• P-validity has no upper bound.
• Both p-validity and coherence are deductive constraints.
• But whereas p-validity applies only to deductive inferences, the scope of coherence is more general.
Coherence and p-validity

• The threshold for p-validity corresponds to the lower bound of the interval for coherence.
• P-validity has no upper bound.
• Both p-validity and coherence are deductive constraints.
• But whereas p-validity applies only to deductive inferences, the scope of coherence is more general. → p-validity enables one to test when people treat deductive and inductive inferences differently.
Measuring coherence

• To what extent are the inferences people make coherent?
• The by far the most studied inferences in psychology: conditional syllogisms

<table>
<thead>
<tr>
<th>MP</th>
<th>MT</th>
<th>AC</th>
<th>DA</th>
</tr>
</thead>
<tbody>
<tr>
<td>If p then q</td>
<td>If p then q</td>
<td>If p then q</td>
<td>If p then q</td>
</tr>
<tr>
<td>p</td>
<td>not-q</td>
<td>q</td>
<td>not-p</td>
</tr>
<tr>
<td>_______</td>
<td>_______</td>
<td>_______</td>
<td>_______</td>
</tr>
<tr>
<td>q</td>
<td>not-p</td>
<td>p</td>
<td>not-q</td>
</tr>
</tbody>
</table>
Please imagine the following situation. Claudia works at blood donation services. She investigates to which blood group the donated blood belongs and whether the donated blood is Rhesus-positive.

Claudia is 60% certain: If the donated blood belongs to the blood group 0, then the donated blood is Rhesus-positive. Claudia knows as well that donated blood belongs with more than 75% certainty to the blood group 0.

How certain should Claudia be that a recent donated blood is Rhesus-positive?
Choice of response format

At least ....% and at most ....% 

Within the bounds of:

0  25  50  75  100  %

Exactly ....% 

Point percentage:

0  25  50  75  100  %

Pfeifer & Kleiter (2007, 2009)
Study 2: Participants' probabilities

If Greece leaves the Euro then Italy will too.
In your opinion, how probable is the above statement/assertion?
Greece will leave the Euro.
In your opinion, how probable is it that the above event occurs?

Singmann, Klauer, & Over (2014)
Study 2: Participants' probabilities

If Greece leaves the Euro then Italy will too.
In your opinion, how probable is the above statement/assertion?
Greece will leave the Euro.
In your opinion, how probable is it that the above event occurs?

If Greece leaves the Euro then Italy will too.
(Probability xx\%)
Greece will leave the Euro.
(Probability xx\%)
Under these premises, how probable is that Italy will leave the Euro, too?

Singmann, Klauer, & Over (2014)
Coherence for MP, lesser for DA

Singmann, Klauer, & Over (2014)
Study 3: Non-numeric responses

\[
\text{AND-elimination:} \quad A \land C \quad \therefore \quad A \\
\text{AND-introduction:} \quad A; \quad C \quad \therefore \quad A \land C \\
\text{OR-introduction:} \quad A \quad \therefore \quad A \lor C \\
\text{AND to IF:} \quad A \land C \quad \therefore \quad \text{IF } A \text{ THEN } C \\
\text{OR to IF-NOT:} \quad A \lor C \quad \therefore \quad \text{IF NOT-} A \text{ THEN } C \\
\text{contraposition:} \quad \text{IF } A \text{ THEN } C \quad \therefore \quad \text{IF NOT-} C \text{ THEN NOT-} A
\]

Politzer & Baratgin (under review)
Scenario

Knowing that the chances are high that now *Nicolas is in Lyon or Jeanne is in Marseille (or both)*, in your opinion, the chances that now *if Nicolas is not in Lyon, Jeanne is in Marseille* are: greater than high; just high; smaller than high.
Coherent above chance level

<table>
<thead>
<tr>
<th>Inference</th>
<th>Coherent in % (chance: 53%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AND elimination</td>
<td>89</td>
</tr>
<tr>
<td>AND introduction</td>
<td>85</td>
</tr>
<tr>
<td>OR introduction</td>
<td>76</td>
</tr>
<tr>
<td>AND to IF</td>
<td>82</td>
</tr>
<tr>
<td>OR to IF-NOT</td>
<td>81</td>
</tr>
<tr>
<td>Contraposition</td>
<td>100</td>
</tr>
</tbody>
</table>

Politzer & Baratgin (under review)
Study 4: Ifs and ands and ors

<table>
<thead>
<tr>
<th>Experiment 1</th>
<th>Experiment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 p, therefore p or q</td>
<td>2.1 p &amp; q, therefore if p then q</td>
</tr>
<tr>
<td>1.2 not-p, therefore not-p or q</td>
<td>2.2 p, q, therefore if p then q</td>
</tr>
<tr>
<td>1.3 If p then q, therefore not-p or q</td>
<td>2.3 p &amp; q, therefore p</td>
</tr>
<tr>
<td>1.4 if not-p then q, therefore p or q</td>
<td>2.4 p &amp; q, therefore q</td>
</tr>
<tr>
<td>1.5 p or q, therefore if not-p then q</td>
<td></td>
</tr>
<tr>
<td>1.6 not-p or q, therefore if p then q</td>
<td></td>
</tr>
</tbody>
</table>

Cruz, Baratgin, Oaksford, & Over (2015)
Now consider the following argument about Linda:

Next to A please indicate how much confidence you would have in the premise of the argument. Next to B please indicate how much confidence you would have in the conclusion, given the premise. Please give a percentage rating from 0% (no confidence at all) to 100% (complete confidence).

A. “Linda votes for the Labour Party or the Green Party”
B. “Therefore, if Linda does not vote for the Labour Party, then she votes for the Green Party”
Exp. 1: Inferences between *if* and *or*

Cruz, Baratgin, Oaksford, & Over (2015)
Exp. 2: Inferences between *if* and *and*

Cruz, Baratgin, Oaksford, & Over (2015)
The meaning of the conditional

- Responses were coherent above chance levels under the assumption that participants interpret the conditional as the conditional event.
- Responses were incoherent above chance levels under the assumption that participants interpret the conditional as material.

Cruz, Baratgin, Oaksford, & Over (2015)
Future directions

• Quantitative measure: not just *whether* responses are coherent or not, but *how* coherent they are
• Boundaries: Under which conditions do people cease to be coherent, and why?
• What role does working memory capacity play for coherence?
Thank you for your attention!
References

References


References


