Measuring coherence in reasoning under uncertainty

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Reasoning under uncertainty: The probabilistic approach

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The conditional event

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- People compute this probability by performing a *Ramsey test* (Evans & Over, 2004; Ramsey, 1929/1994; Stalnaker, 1968).

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p, q	True
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The material conditional

The normative system question

- When investigating reasoning from uncertain premises, we require criteria for the correctness of an inference that take account of uncertainty.

Binary consistency

- In the binary approach to reasoning, a central criterion for the correctness of an inference was given by whether the statements involved in the inference were consistent or not: The absence of a contradiction.

Binary validity

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$\rightarrow$ binary validity is truth preserving.

Probabilistic criteria for inference correctness

- A central development in the probabilistic approach was the generalisation binary consistency to **coherence**, and the generalisation of binary validity to probabilistic validity, **p-validity**.

Coherence

- An inference is coherent when it complies with the axioms of probability theory (de Finetti, 1936).

$$P(A) = .6$$

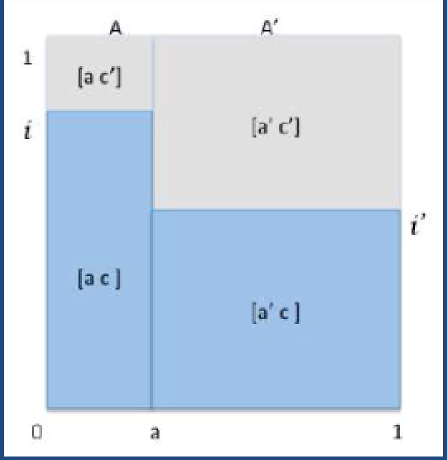
$$\Rightarrow \quad P(\text{not-}A) = .4$$

Justifications for coherence: Dutch Books

- If a person acts in an incoherent way, then a Dutch book can be made against her: A series of bets that are guaranteed to lead to a net loss for her, independently of the outcome of the bets (de Finetti, 1936; Ramsey, 1926/1994).

Justifications for coherence: Water tank analogy

- Also, if a person acts incoherently, this is as if she would pour liquid into compartments of a tank in a way that violates physical laws (Politzer, 2014).



Intervals for coherence

- Given the probabilities of the premises, the conclusion is coherent if it falls within a certain probability interval.
- If the premises are very informative for the conclusion, the interval can reduce to a point value.
- If the premises are non-informative for the conclusion, the interval extends to the whole probability range.

Coherence: Example

- Linda is a feminist and a bank teller ____%
- **Therefore**, Linda is a bank teller. ____%

Tvertsky & Kahneman (1983)

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$$P(B) \in [P(F \& B), 1]$$

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P-validity

- Let the uncertainty of a statement equal 1 minus its probability: U(A) = 1 – P(A).
- Then an inference is p-valid iff there are no coherent assignments of probabilities to the premises and conclusion in which the uncertainty of the conclusion is greater than the sum of the uncertainties of the premises (Adams, 1998).

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$\rightarrow$ p-validity enables one to test when people treat deductive and inductive inferences differently.

Measuring coherence

- To what extent are the inferences people make coherent?
- The by far the most studied inferences in psychology: conditional syllogisms

MP	MT	AC	DA
If p then q	If p then q	If p then q	If p then q
p	not-q	q	not-p
q	not-p	p	not-q

Study 1 - Explicit intervals

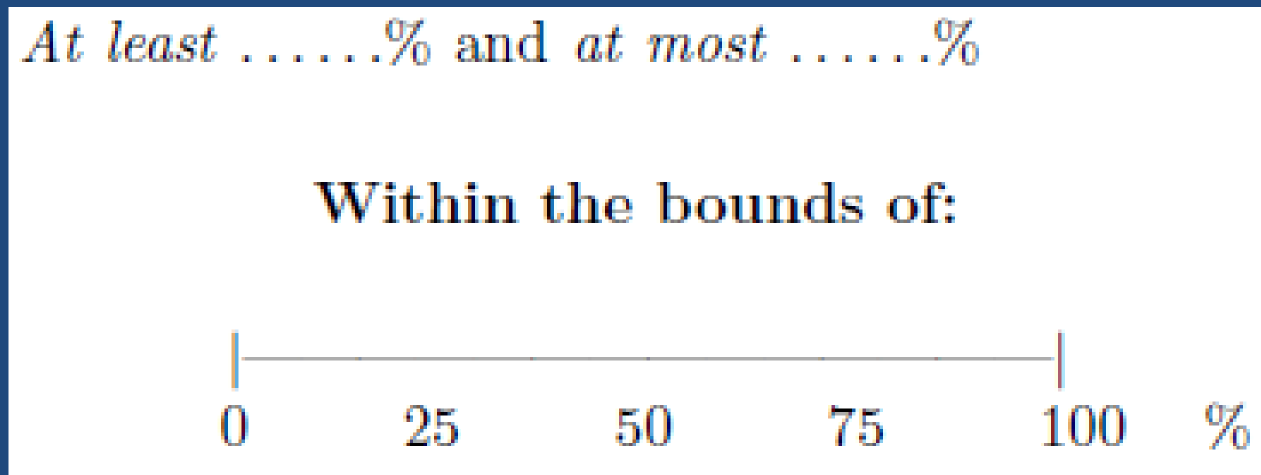
Please imagine the following situation. Claudia works at blood donation services. She investigates to which blood group the donated blood belongs and whether the donated blood is Rhesus-positive.

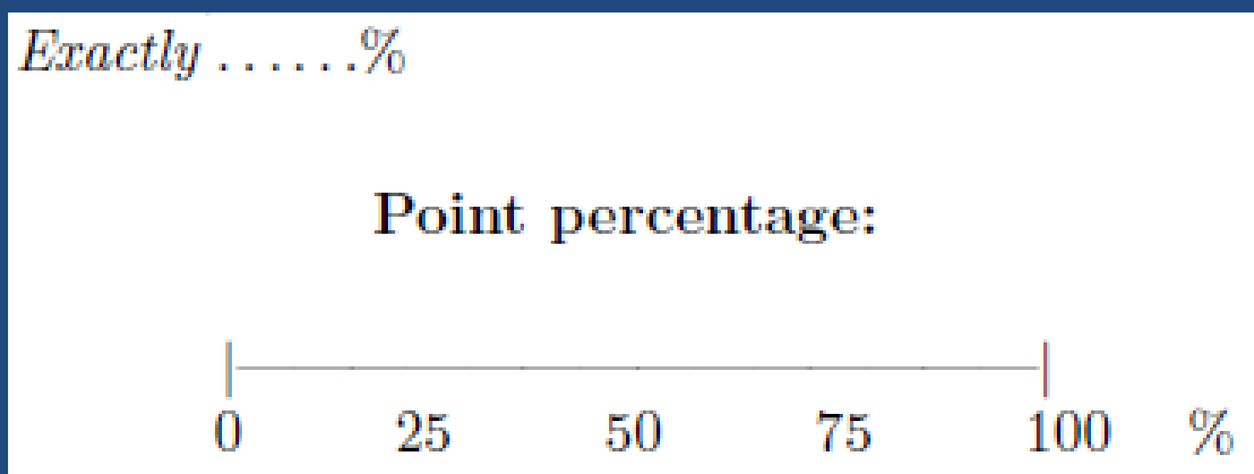
> Claudia is 60% certain: If the donated blood belongs to the blood group 0, then the donated blood is Rhesus-positive. Claudia knows as well that donated blood belongs with *more than* 75% certainty to the blood group 0.

How certain should Claudia be that a recent donated blood is Rhesus-positive?

Pfeifer & Kleiter (2007, 2009)

Choice of response format





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Study 2: Participants' probabilities

If Greece leaves the Euro then Italy will too.

In your opinion, how probable is the above statement/assertion?

Greece will leave the Euro.

In your opinion, how probable is it that the above event occurs?

Singmann, Klauer, & Over (2014)

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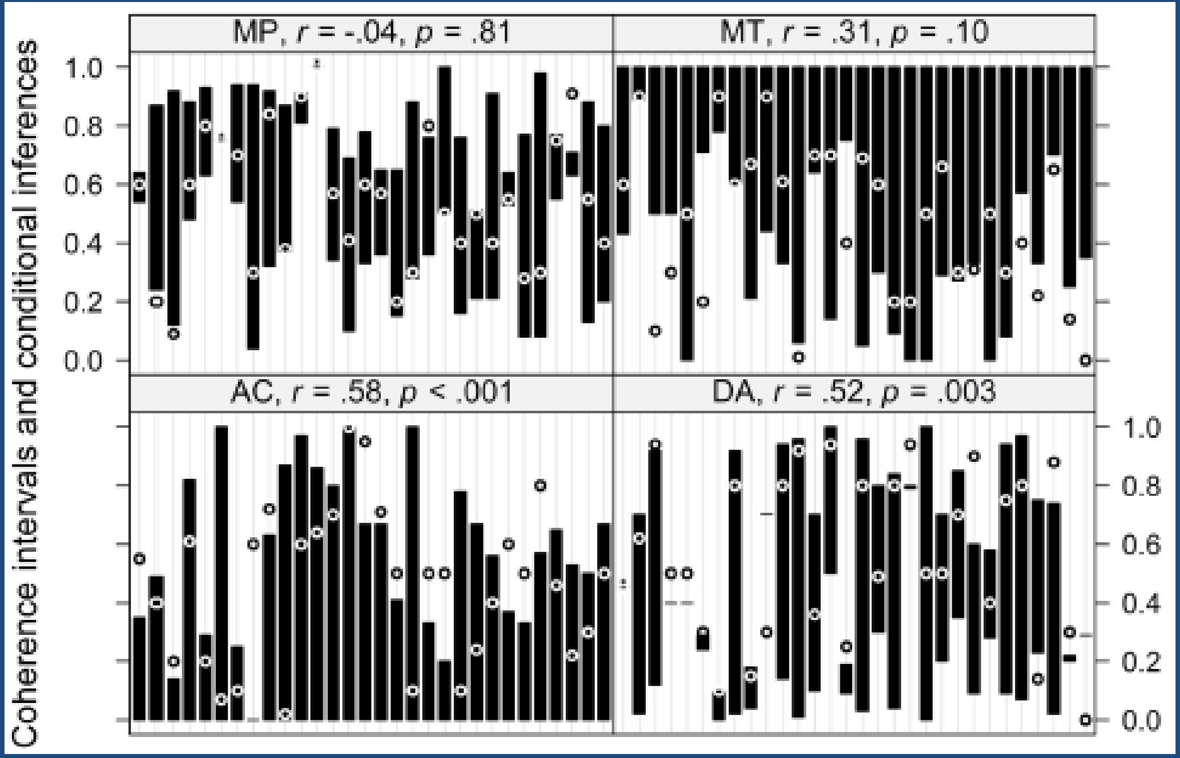
If Greece leaves the Euro then Italy will too.
(Probability xx%)
Greece will leave the Euro.
(Probability xx%)
Under these premises, how probable is that Italy will leave the Euro, too?

Singmann, Klauer, & Over (2014)

Coherence for MP, lesser for DA

Percentage of coherent responses/coherent responses predicted by chance.

MP	MT	AC	DA
87%/45%	63%/65%	60%/58%	60%/46%



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Study 3: Non-numeric responses

AND-elimination:	A AND C ∴ A
AND-introduction:	A; C ∴ A AND C
OR-introduction:	A ∴ A OR C
AND to IF:	A AND C ∴ IF A THEN C
OR to IF-NOT:	A OR C ∴ IF NOT-A THEN C
contraposition:	IF A THEN C ∴ IF NOT-C THEN NOT-A

Politzer & Baratgin (under review)

Scenario

Knowing that the chances are high that now *Nicolas is in Lyon or Jeanne is in Marseille (or both)*, in your opinion, the chances that now *if Nicolas is not in Lyon, Jeanne is in Marseille* are: greater than high; just high; smaller than high.

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Coherent above chance level

Inference	Coherent in % (chance: 53%)
AND elimination	89
AND introduction	85
OR introduction	76
AND to IF	82
OR to IF-NOT	81
Contraposition	100

Politzer & Baratgin (under review)

Study 4: Ifs and ands and ors

Experiment 1		Experiment 2	
1.1	p, therefore p or q	2.1	p & q, therefore if p then q
1.2	not-p, therefore not-p or q	2.2	p, q, therefore if p then q
1.3	If p then q, therefore not-p or q	2.3	p & q, therefore p
1.4	if not-p then q, therefore p or q	2.4	p & q, therefore q
1.5	p or q, therefore if not-p then q		
1.6	not-p or q, therefore if p then q		

Cruz, Baratgin, Oaksford, & Over (2015)

Participants' probabilities

Now consider the following argument about Linda:

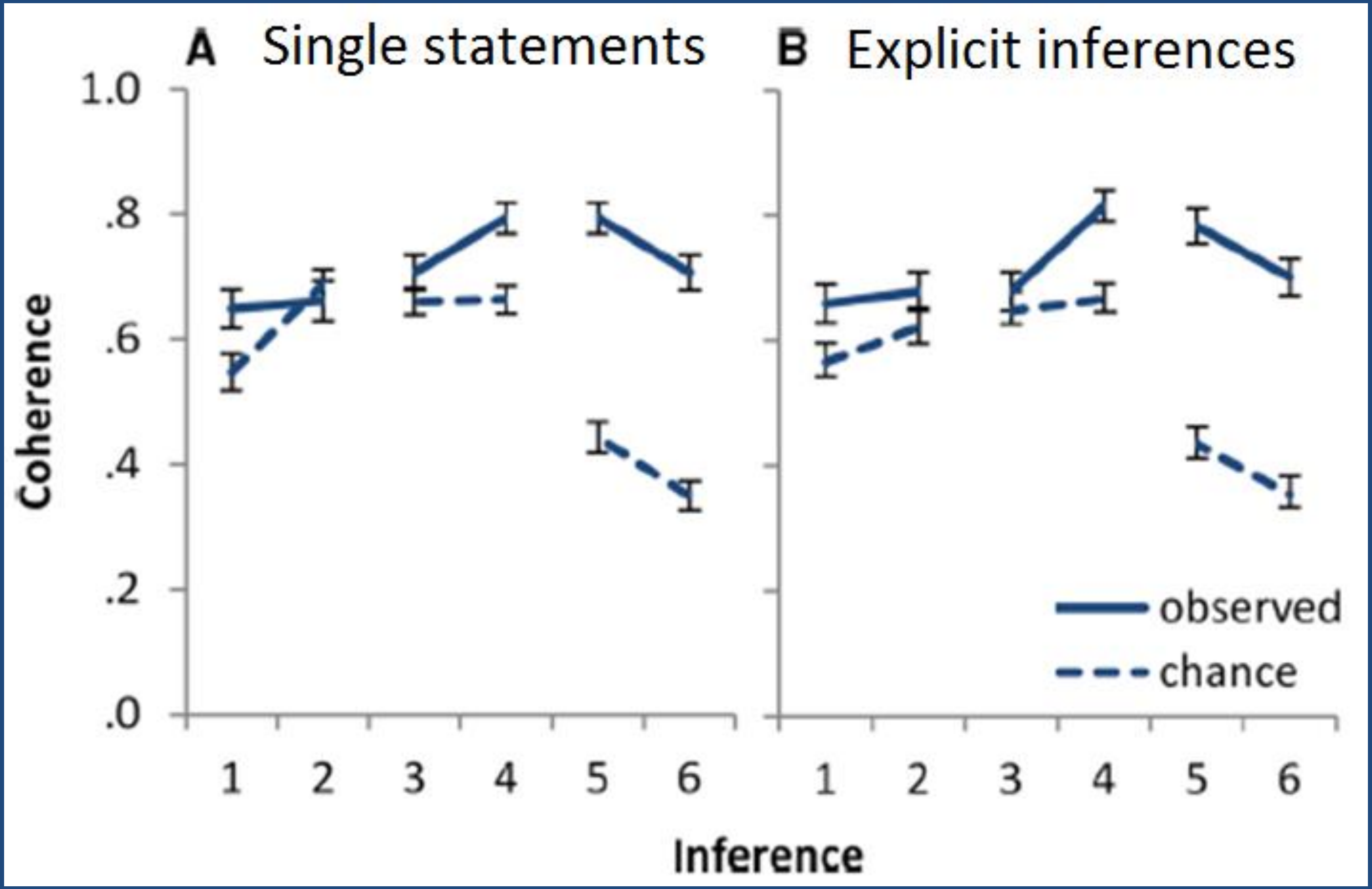
Next to A please indicate how much confidence you would have in the premise of the argument. Next to B please indicate how much confidence you would have in the conclusion, given the premise. Please give a percentage rating from 0% (no confidence at all) to 100% (complete confidence).

A. "Linda votes for the Labour Party or the Green Party"

B. "Therefore, if Linda does not vote for the Labour Party, then she votes for the Green Party"

Cruz, Baratgin, Oaksford, & Over (2015)

Exp. 1: Inferences between *if* and *or*



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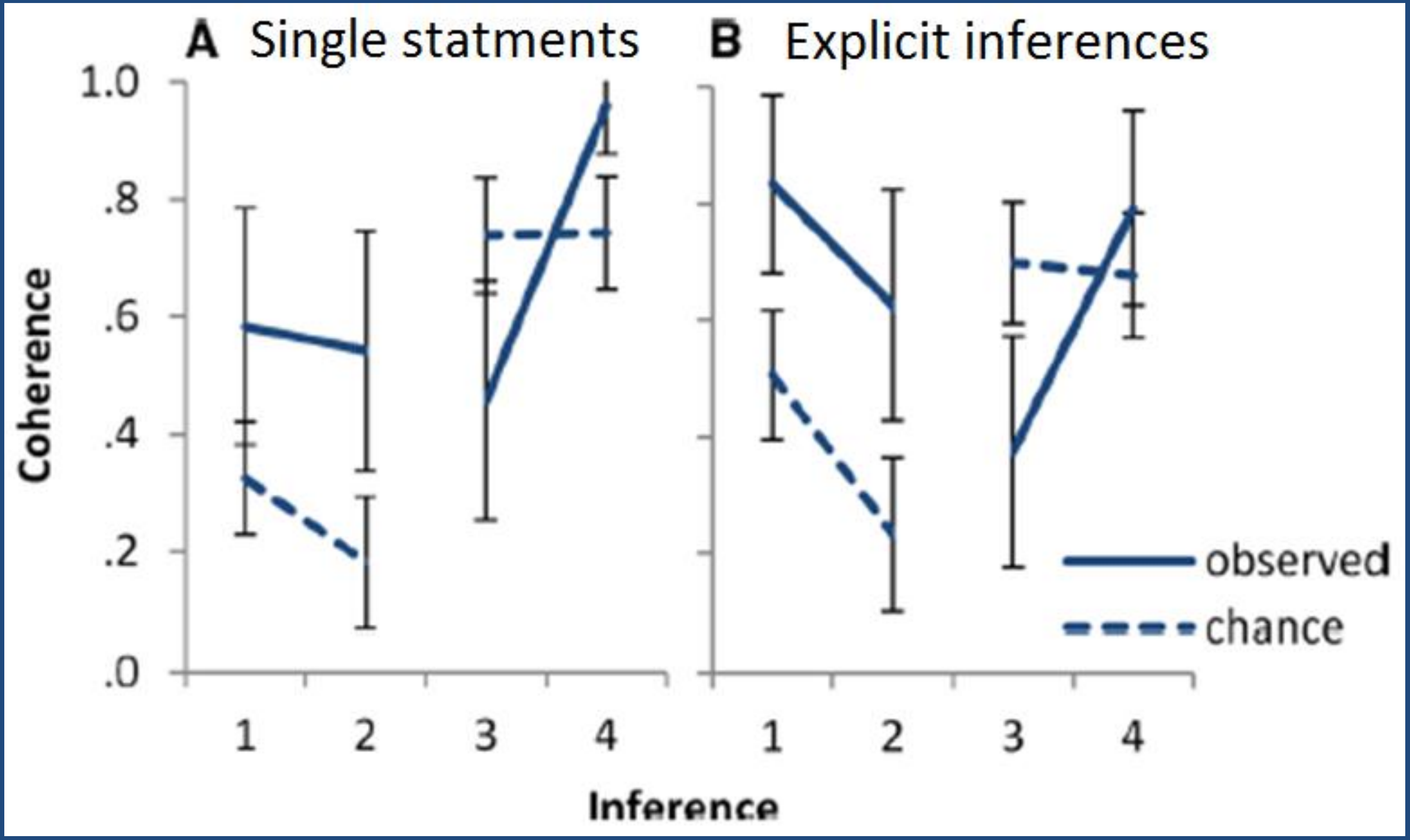
Exp. 2: Inferences between *if* and *and*



Cruz, Baratgin, Oaksford, & Over (2015)

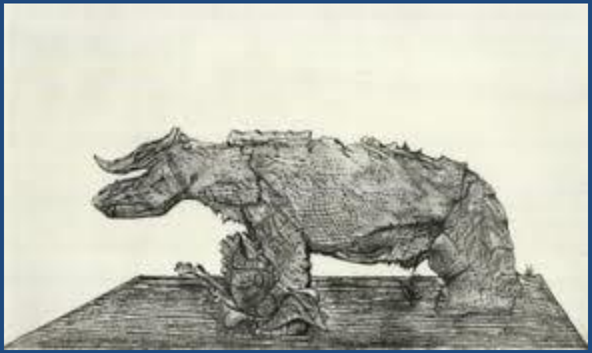
The meaning of the conditional

- Responses were coherent above chance levels under the assumption that participants interpret the conditional as the conditional event
- Responses were **in**coherent above chance levels under the assumption that participants interpret the conditional as material.

Cruz, Baratgin, Oaksford, & Over (2015)

Future directions

- Quantitative measure: not just *whether* responses are coherent or not, but *how* coherent they are
- Boundaries: Under which conditions do people cease to be coherent, and why?
- What role does working memory capacity play for coherence?



Thank you for your attention!

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