The “defective” truth table: Its past, present, and future

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The material conditional truth table

If you buy this lottery ticket (p), you will win some money (q)

1 = true, 0 = false

q / *p*	1	0
1	1	0
0	1	1

The so-called "defective" truth table

If you buy this lottery ticket (p), you will win some money (q)

1 = true, 0 = false, U = uncertain

p \ *q*	1	0
1	1	0
0	U	U

Wason (1966): The first reference

If a card has a vowel on one side then it has an even number on the other side.

"Subjects assume implicitly that a conditional statement has, not two truth values, but three: true, false, and 'irrelevant'. Vowels with even numbers verify, vowels with odd numbers falsify, and consonants with any number are irrelevant." (p. 146)

Note three points. First the term *defective* is not used. Second the above conditional seems to be a general conditional and not a singular conditional. Third *irrelevant*, or rather *'irrelevant'*, is said to be a truth value of the conditional and also a property of some of the cards.

Johnson-Laird & Tagart (1969)

This is the first extended paper on the "defective" truth table, and the first time it is displayed as a table. But again the term *defective* is not used.

They say a conditional is neither true nor false but is itself *irrelevant* when its antecedent is false.

They also state, "The S's task was to consider each stimulus in turn and to decide whether it indicated that the sentence was true or false, or was irrelevant to the truth-value of the sentence."

Their conditionals are of the form, *If there is an A on the left, then there is a 7 on the right*, and so seem to be general conditionals.

Ambiguities and uncertainties

There are a number of ambiguities and uncertainties in the literature on the "defective" truth table. We have seen two of them right at the beginning of the literature on it.

Is the conditional singular or general?

Does *irrelevant* refer to a third truth value and thus to a semantic property of a conditional, or does it refer to a pragmatic or semantic property of the objects referred to by the conditional?

First ambiguity

Evans, Handley, & Over (2003) distinguished probability judgments about <u>singular</u> conditionals from those about <u>general</u> conditionals, asking "How likely is the following claim to be true (false) of a card drawn at random?"

If the card is yellow then it has a circle printed on it.

The above is a singular conditional. A general conditional, "If a (or any) card is yellow …", does not necessarily have some third, "irrelevant", truth value when a card drawn at random turns out to be red.

Second ambiguity

Suppose a card is to be drawn at random and "the card" in the following refers to it:

If the card is yellow then it has a circle printed on it.

Many objects are "irrelevant" to this conditional in the ordinary sense. Take any red herring you like.

But now suppose that the selected card is red. Does that imply this conditional is neither true nor false, but has a third value of some type?

Use of the term *defective*

Wason (1966) does not use *defective* for this truth table at all, nor do Johnson-Laird & Tagart (1969), though the later refer to Kneale & Kneale (1962), who do use it and are the first authors to do so, as far as I know.

Johnson-Laird & Wason (1970) use the term just once and in passing.

Evans (1972) uses it in his Abstract and then a number of times in an in-depth study, and Wason & Johnson-Laird (1972) use it in their book and base what they say about it on Quine (1952).

Kneale & Kneale (1962)

Conditional utterances are compared: A conditional prediction, a conditional promise, and a conditional order.

Kneale & Kneale hold that, in false antecedent cases, the prediction is not verified, the promise is not kept, and the order is not obeyed.

The truth table is "defective". <u>No</u> truth value, and <u>not</u> some third truth value, is assigned in false antecedent cases. The table is displayed.

Wason & Johnson-Laird (1972)

Kneale & Kneale do not base their account on Quine, but Wason & Johnson-Laird do, concluding, "Hence, as Quine (1952) remarked, the ordinary conditional is a conditional assertion rather than the assertion of the conditional … On this presuppositional analysis, the conditional has an incomplete truth table: no value is specified for those cases where antecedent is false … "

The "defective" truth table then follows, with "void" used as the third "value".

Newstead et al. (1997)

Newstead, Ellis, Evans, & Dennis (1997, T&R). More attention should be given to this paper. They study a number of different conditional "speech acts".

There are promises, threats, tips, and warnings. But unfortunately, the participants are always asked if the conditional is "supported", "contradicted", or one can learn "nothing about" it.

It can be argued that one does "learn something" in false antecedent cases.

Conditional bets

Both Ramsey (1926, 1929) and de Finetti (1936) compared conditional assertions to conditional bets, like one on:

I bet you £10 that, if the card is yellow, then it has a circle printed on it.

The bet is won, and the conditional assertion true, when the selected card is yellow and has a circle on it. It is lost, and the assertion is false, when the card is yellow and does not have a circle on it. The conditional bet is "void", and so is the conditional assertion, when the card is not yellow.

Politzer et al. (2010) confirmed this parallel relationship.

What does "void" mean?

It is often said that nothing at all has been expressed if a conditional speech act has a false antecedent and is "void" (Quine, 1952) . But this cannot be right.

A "void" conditional states no indicative fact. It is about a hypothetical possibility.

In traditional truth table tasks, people are actually shown a false antecedent case. The indicative conditional is then "void". People would not use an indicative form in such a case. They would use a counterfactual.

A void case in more detail

I say to an editor, "If you publish their paper, it will be the worst mistake you ever make." The editor rejects the paper, but it is published elsewhere to great acclaim.

I can hardly justify myself by claiming that what I said was a material conditional and so true! Equally I cannot claim to have made a "void" utterance in every sense of this term. It was about a hypothetical possibility.

There are corresponding counterfactuals, "If you were to accept the paper, …", and "If you had accepted it, …", These can have a high or low probability and be more or less justified.

The Ramsey test

The Ramsey test has to be added to Quine's account of conditional assertions:

Ramsey (1931): People judge *if p then q* by "...adding *p* hypothetically to their stock of knowledge …" and then assessing to what extent *q* follows. They thus fix "... their degree of belief in q given p …", P(q|p).

The Ramsey test is used to evaluate a conditional when its antecedent is not known to be true. It is not needed when *p* is known to be true.

The Ramsey test and the truth table task

In a traditional task, people can see that *p* is true and *q* is true in the first row, and that *p* is true and *q* is false in the second row.

In the third and fourth rows, *p* is seen to be false. Now this entry is "irrelevant". There is <u>uncertainty</u> in these cases, and the Ramsey test must be used to resolve it to some degree.

In these rows, the indicative is "void", but if people have background knowledge, they can use the Ramsey test to evaluate the corresponding <u>counterfactual</u>.

The de Finetti table

The "defective" truth table should be called the *de Finetti Table*, or more strictly the *2x2 de Finetti table*.

This table was first proposed by de Finetti in (1936), in his paper on "The logic of probability". This table is part of a probabilistic account of reasoning and the conditional, in which P(if p then q) = P(q|p).

He called this conditional the *conditional event* and related it to a conditional bet, which is basically a bet on a conditional assertion.

The 2x2 de Finetti table for *if p then q*

1 = true, 0 = false, U = uncertain

q / *p*	1	0
1	1	0
0	U	U

The 3x3 general de Finetti table

Clearly, people do not always know which row of the truth table represents the actual state of affairs.

Which three-value table best represents people's judgments when they are uncertain about *p* or about *q*? Many three-value tables have been proposed. Which best describes the judgments of ordinary people, de Finetti's or another?

The 3x3 de Finetti table (Baratgin et al., 2013)

1 = certainly true, 0 = certainly false, U = uncertain

p \ *q*	1	U	0
1	1	U	0
U	U	U	U
0	U	U	U

The future of truth table studies

What is the future of truth table studies?

In the 3x3 table, the “third” value is “uncertainty”, but of course this is not fully general.

Still more generally, “uncertainty” is replaced by degrees of belief / subjective probability.

Introducing the Jeffrey table

Baratgin et al. (2013) point out that the use of "uncertain" in the 3x3 table can be divided into the varying degrees of subjective probability.

The conditional probability, P(q|p), can itself be entered as the value of *if p then q*. Such a table can be called a *Jeffrey table* (after Jeffrey, 1991).

In a Jeffrey table, the factual and subjective uses of "true" and "false" are found in one table. Sometimes *if p then q* is "true" as a result of a *p & q* state of affairs. But when *p* is false, *if p then q* expresses the subjective probability P(q|p), and of course P(q|p) will be 1 in some cases.

The Jeffrey table derived

Much research has confirmed that the probability of *if p then q* is judged to be the conditional probability, P(q|p).

What value replaces the U of "uncertainty" in the table if P(if p then q) = P(q|p)? Jeffrey (1991) showed that:

$$P(q|p) = P(p \& q)(1) + P(p \& \text{not-}q)(0) + P(\text{not-}p)U$$
$$P(q|p) = P(p \& q) + P(\text{not-}p)U$$
$$P(q|p) - P(p \& q) = P(\text{not-}p)U$$
$$P(q|p) – (P(p)P(q|p)) = P(\text{not-}p)U$$
$$P(q|p)(1 – P(p)) = (1 – P(p))U$$
$$P(q|p) = U$$

The Jeffrey truth table for *if p then q*

1 = certainly true, 0 = certainly false,

P(q|p) = the conditional probability of *q* given *p*

p \ *q*	1	0
1	1	0
0	P(q|p)	P(q|p)

The Jeffrey table for an example in Byrne & Johnson-Laird (2009)

If Obama wins the election (o), the Republicans will have lost (r)

o \ ***r***	**1**	**0**
1	**1**	**0**
0	**1**	**1**

The Jeffrey table against Byrne & Johnson-Laird

If Obama wins the election (o), the Democrats will have lost (r)

d / *o*	1	0
1	1	0
0	0	0

The Jeffrey table for *if p then p*

p	*if p then p*
1	1
0	1

The Jeffrey table: Further points

- The Jeffrey table displays the results of the Ramsey test in the table itself.
- A logical truth, e.g. “if p then p”, is never “uncertain” in the table, but has a value of 1.
- The Jeffrey table integrates well with the new paradigm accounts of the validity of inferences from beliefs and the consistency of beliefs.
- The problem is that it is difficult to see how to extend the Jeffrey table to iterated, compound conditionals.

Final questions about truth

- Is *true* univocal or does it have more than one meaning, semantic, pragmatic ... ?
- Do ordinary people use *true* in more than one way?
- Case for saying that de Finetti thought of *true* as having one meaning, but that was having a probability of 1.
- Do ordinary people always treat *true* as equivalent to having a probability of 1? Do they sometimes treat the two as equivalent?