Conditional reasoning and probabilistic validity

David Over
Psychology, Durham University

Financial support for part of the research, with Jean Baratgin and Guy Politzer (Jean Nicod), in this talk was provided by the ANR agency under grant ANR Chorus 2011 (project BTAFDOC). Also collaborating on parts of this research with Jonathan Evans and Valerie Thompson. And thanks to Christoph Klauer, Gernot Kleiter, Niki Pfeifer and Henrik Singmann,

The old paradigm: An example

If Linda is at the party, she is drinking too much.
She is at the party.
Therefore, she is drinking too much.

In the old paradigm, participants in an experiment on this instance of MP would be asked to <u>assume</u> the premises, treating them in effect as certain, to <u>set aside</u> relevant beliefs they might have, and then to say whether or not the conclusion <u>necessarily</u> followed. The conditional *if p then q* was held to be the material conditional, *not-p or q*.

A difficult question in the old paradigm

Assume:

No dog is a mammal.
Some cat is a dog.

Consider this conclusion:

Some cat is not a mammal

Does it necessarily follow?

The old paradigm: Basic points

The older paradigm distinctions are binary. Propositions are to be assumed or not assumed as premises and are represented as only either true or false or as consistent or inconsistent . Conclusions follow necessarily or not at all.

The conditional is the material conditional, *not-p or q*, and its extensional logic is the relevant normative theory

The definition of logical validity in the old paradigm is about the preservation of truth in extensional logic from assumed premises. Failure to conform to this logic is a "fallacy".

The new paradigm: Basic points

There is a new Bayesian, or probabilistic, paradigm in the psychology of reasoning. Human inference is not from arbitrary assumptions, but from beliefs that are usually uncertain to some degree.

The relevant normative theory in the new approach is the logic of partial belief (Ramsey) / the logic of subjective probability or uncertainty (de Finetti), and its account of human rationality is Bayesian / probabilistic.

The new paradigm: An example

If Linda goes to the party, she will drink too much.
She will go to the party.
Therefore, she will drink too much.

The new paradigm recognizes that most inference, whether in everyday or scientific reasoning, is from premises or <u>beliefs</u> that are <u>uncertain</u> and cannot be simply assumed true. Linda does not always drink too much at parties, and there is a chance she will not go to the party. The question is, "How likely is she to drink too much at the party?"

Rational decision making

"If I buy a UK lottery ticket (p), then I will win millions (q)."

In the old paradigm, *if p then q* is equivalent to *not-p or q*, and so P(if p then q) = P(not-p or q).

But being rational I decide against buying a ticket, making P(not-p) high, so making P(not-p or q) high and supposedly P(if p then q) high. But how can I then be rational?

The new paradigm integrates reasoning and decision making much more successfully.

The new paradigm: Logical validity

A probabilistic definition of logical validity can be given in the new paradigm, called p-validity (Adams, 1998).

An informal definition is that an inference is p-valid if it preserves probability and not just certain truth.

More formally, an single premise inference is p-valid if and only if the probability of its premise cannot be coherently higher than the probability of its conclusion.

The conjunction fallacy

Tversky & Kahneman (1983) pointed out the fallacy of judging that P(f & t) > P(t). *f & t* "logically implies" *t*, and so P(f & t) cannot be coherently greater than P(t).

We can turn this round and define "logical validity" in terms of probability judgments. A single premise inference is <u>p-valid</u> if (and only if) the probability of the premise cannot be coherently greater than the probability of the conclusion.

This definition allows us to refer to the logical validity of inferences from degrees of belief .

The heart attack example 1

Let *f* now be the statement that Linda is over 55 years old and *t* be the statement that she has had a heart attack.

Suppose people's degree of belief in *f & t* is .1, and then they infer a degree of belief that *t* holds. When they actually perform this inference, called "& - elimination", will they conform to p-validity? Will they coherently hold that P(t) ≥ P(f & t)?

But note this problem: people can conform by "chance" to p-validity. In this case, the probability is .9 that this will happen, 1 – P(f & t).

The heart attack example 2

Let f now be the statement that Linda is over 55 years old and t be the statement that she has had a heart attack.

Suppose people' s degree of belief P(f & t) = .1, and they infer a degree of belief P(t) = .6.

So far it appears that they have conformed to p-validity, but if they now go on to infer that P(f) = .6, then they are incoherent. P(f & t) = .1 is too low.

The new paradigm, using p-validity, sets upper and lower bounds on human reasoning (Pfeifer & Kleiter).

The heart attack example 3

To avoid the *extended* conjunction fallacy, P(f & t) must be in an interval. Where P(f) = x and P(t) = y, this is:

$$\min(x, y) \geq P(f \,\&\, t) \geq \max(0, x + y - 1)$$

For our example, where P(f) = P(t) = .6, the interval is:

$$.6 \geq P(f \,\&\, t) \geq .2$$

Again there is a chance of being correct with a "guess" about P(f & t). In our example, this is .4. Note also that Jonathan Evans has developed a way of accounting for "guessing" in research with Valerie Thompson and me.

The new paradigm and the conditional

The natural language indicative conditional is held to be the <u>probability conditional</u> (Adams) or the <u>conditional event</u> (de Finetti). Its probability is the conditional probability:

$$P(\text{if } p \text{ then } q) = P(q|p)$$

The above is sometimes called <u>the Equation</u> both in philosophy and psychology (Edgington, 1995; Oaksford & Chater, 2007, 2009). It is so called because of its fundamental implications. Once it is confirmed as a descriptive statement of human judgment, a <u>Bayesian account of human reasoning</u> will follow (Oaksford & Chater, 2007).

Introducing conditionals: The Ramsey test

Ramsey (1931): People can assess *if p then q* by "...adding p <u>hypothetically</u> to their stock of knowledge …" They would thus fix "...their degrees of belief in *q* given *p*…", P(q|p).

In Ramsey's original example, the two people were arguing about *if p then q*, and so there could be a winner and a loser in the debate.

The basic de Finetti / "defective" truth table for *if p then q*

T = true, F = false, W = win, L = lose, V = void

q / *p*	T	F
T	T (W)	F (L)
F	V	V

The Ramsey test and de Finetti table

<u>The Ramsey test</u> and <u>de Finetti table</u> are the pillars that support Bayesian, or probabilistic, accounts of human conditional reasoning. In such accounts, indicative conditionals, conditional bets, and the conditional probability should all be closely related to each other.

The Equation as the conditional probability hypothesis

The conditional probability hypothesis, that P(if p then q) = P(q|p) , has been highly confirmed in recent experiments (including Evans et al., 2003; Over et al, 2007; Douven & Verbrugge, 2010; Fugard et al., 2011). Versions of the new paradigm most relevant to this talk are those of Evans & Over (2004), Oaksford & Chater (2007), Politzer, Over, & Baratgin (2010), and Pfeifer & Kleiter (2010).

Generalizing p-validity: More than one premise

P(p & q) cannot be coherently greater than P(p).

More generally, the probabilities of the premises of a valid inference cannot be coherently greater than the probability of the conclusion.

More generally and more precisely, when P(if p then q) = P(q|p), let the uncertainty of any premise or conclusion *s* be 1 - P(s). Then an inference is <u>p-valid</u> if (and only if) the uncertainty of its conclusion cannot be coherently greater than the sum of the uncertainties of its premises.

Suppression is not necessarily a bias

If Linda goes to the party, she will drink too much.
If there is wine at the party, Linda will drink too much.
She will go to the party.
Therefore, Linda will drink too much.

The new paradigm explains why the extra premise causes a loss of confidence in the conclusion of MP above.

“Suppression” is not a “fallacy” as long as uncertainty about the conclusion is not too great.

The central example of a two premise inference: MP

If Linda goes to the party (p), then she will drink too much (q). She will go to the party. Therefore, she will drink too much.

Where $P(q|p) = .9$ and $P(p) = .8$, $P(q)$ should be by total probability:

$$P(q) = P(p)P(q|p) + P(\text{not-}p)P(q|\text{not-}p) = .72 + .2P(q|\text{not-}p)$$

Clearly, MP is <u>p-valid</u>. Moreover, $P(q|\text{not-}p)$ will be 0 at the minimum and 1 at the maximum, $P(q)$ should fall in the interval [.72 , .92] for <u>coherence</u>.

If people claim that $P(q) < .72$ or $P(q) > .92$, they are committing a fallacy. This is like the Linda fallacy at a general level. The old paradigm cannot even express the idea that confidence in the conclusion of MP can be <u>too high</u>.

Conforming to p-validity and the interval by chance

If Linda goes to the party (p), she will drink too much (q). She will go to the party. Therefore, she will drink too much.

Where $P(q|p) = .9$ and $P(p) = .8$, $P(q)$ should be by total probability:

$$P(q) = P(p)P(q|p) + P(\text{not-}p)P(q|\text{not-}p) = .72 + .2P(q|\text{not-}p)$$

The sum of the uncertainties of the premises is $.1 + .2 = .3$. To conform to p-validity, we must judge $P(q) \geq .7$, as $1 - .3 = .7$. The chance that we will be correct by guessing is .3.

To be coherent, we must judge $.72 \leq P(q) \leq .92$. The chance that we will be coherent by guessing is $.92 - .72 = .2$.

The old paradigm as a special case

If Linda is at the party (p), she is drinking too much (q). She is at the party. Therefore, she is drinking too much.

The old paradigm can be seen as a special case of the new paradigm, one in which the probabilities are either 1 or 0. Let P(q|p) = 1 and P(p) = 1. Then P(q) must be 1

$$P(q) = P(p)P(q|p) + P(\text{not-}p)P(q|\text{not-}p) = 1 + 0 = 1$$

Some people of relatively high cognitive ability and special training are able to use higher level processes - Type 2 in dual process theory - to make arbitrary assumptions and hold these as certain for the purposes of an inference.

The logic of probability: Point values

If Linda goes to the party (p), she will drink too much (q). She will go to the party. Therefore, she will drink too much.

Let $P(q|p) = .9$, $P(p) = .8$, $P(q|\text{not-}p) = .05$

$P(q) = P(p)P(q|p) + P(\text{not-}p)P(q|\text{not-}p) = .72 + (.2)(.05) = .73$

With the above degrees of belief as premises, how close will people be to the point value degree of belief for the conclusion?

Research on this is just beginning (Zhao & Osherson, 2010, and in work that Dinos Hadjichristidis, Steven Sloman, and I are doing).

Conditional inference task

If it is sunny then it is hot

Modus Ponens (MP) Valid

It is sunny; therefore it is hot

Modus Tollens (MT) Valid

It is not hot; therefore it is not sunny

Affirmation of the Consequent (AC) Invalid

It is hot; therefore it is sunny

Denial of the Antecedent (DA) Invalid

It is not sunny, therefore it is not hot

Other conditional inferences

The valid MT - inferring *not-p* from *if p then q* and *not-q* - is not as highly endorsed as MP.

The invalid AC - inferring *p* from *if p then q* and *q* – nnd the invalid DA - inferring *not-q* from *if p then q* and *not-p* - are quite highly endorsed.

These appeared to be fallacies in the old paradigm.

The other inferences and p-validity

The conclusion of MT can be uncertain because the minor premise - *not-q* - can make the major premise uncertain, although MT is p-valid.

AC and DA are p-<u>in</u>valid but can be <u>strong</u> inferences in the new paradigm.

Strong and forceful inferences

An inference is strong to the extent that its conclusion is probable given degrees of belief in its premises.

An inference is forceful to the extent that it leads to a large change in the degree of belief in its conclusion.

Many inferences condemned as “fallacies” by the old paradigm can be seen as strong or forceful inferences in many contexts in a Bayesian analysis (Ulrike Hahn, Mike Oaksford, Adam Harris, and others.)

Do people respect p-validity and coherence in their conditional beliefs and inferences?

- **One way to get an answer is simply to ask people to rate their beliefs in *if p then q, p, q, not-p,* for a particular set of realistic conditionals.**

- **We can then compare the probabilities assigned for p-validity and coherence.**

- **This measures <u>implicit</u> p-validity and coherence in their beliefs and is a very strong test of the Bayesian requirement that subjective probabilities conform with the probability theory.**

Explicit reasoning and dual process theory

- **Dual process theory** holds that many degrees of belief are formed by implicit heuristic, **type 1**, processes, but explicit reasoning is a **type 2** process that allows the use of logical or other rules in working memory.
- When beliefs are grouped together to indicate their inferential structure, type 2 processes can use logical and coherence relations in probability judgments.
- Such grouping tests whether p-validity and coherence can be achieved with explicit reasoning effort.

EXPERIMENT

Jonathan Evans, Valerie Thompson, & David Over

- **Have run a small study as a first step, and constructed 48 realistic conditional sentences expressing causal or temporal relations about events in the near future, e.g. If more houses are built, then the number of homeless people will decrease/increase.**

- **The study was run in Saskatoon and statements contextualised for Canadian students.**

Belief group (n = 23)

- **Rated one list of 48 conditionals, *if p then q,* assigning probabilities to represent beliefs in randomized order.**

- **Rated a second event list containing both affirmative and negative statements representing *p, not-p, q***
- **and *not-q* to make minor premises and conclusions for inferences.**

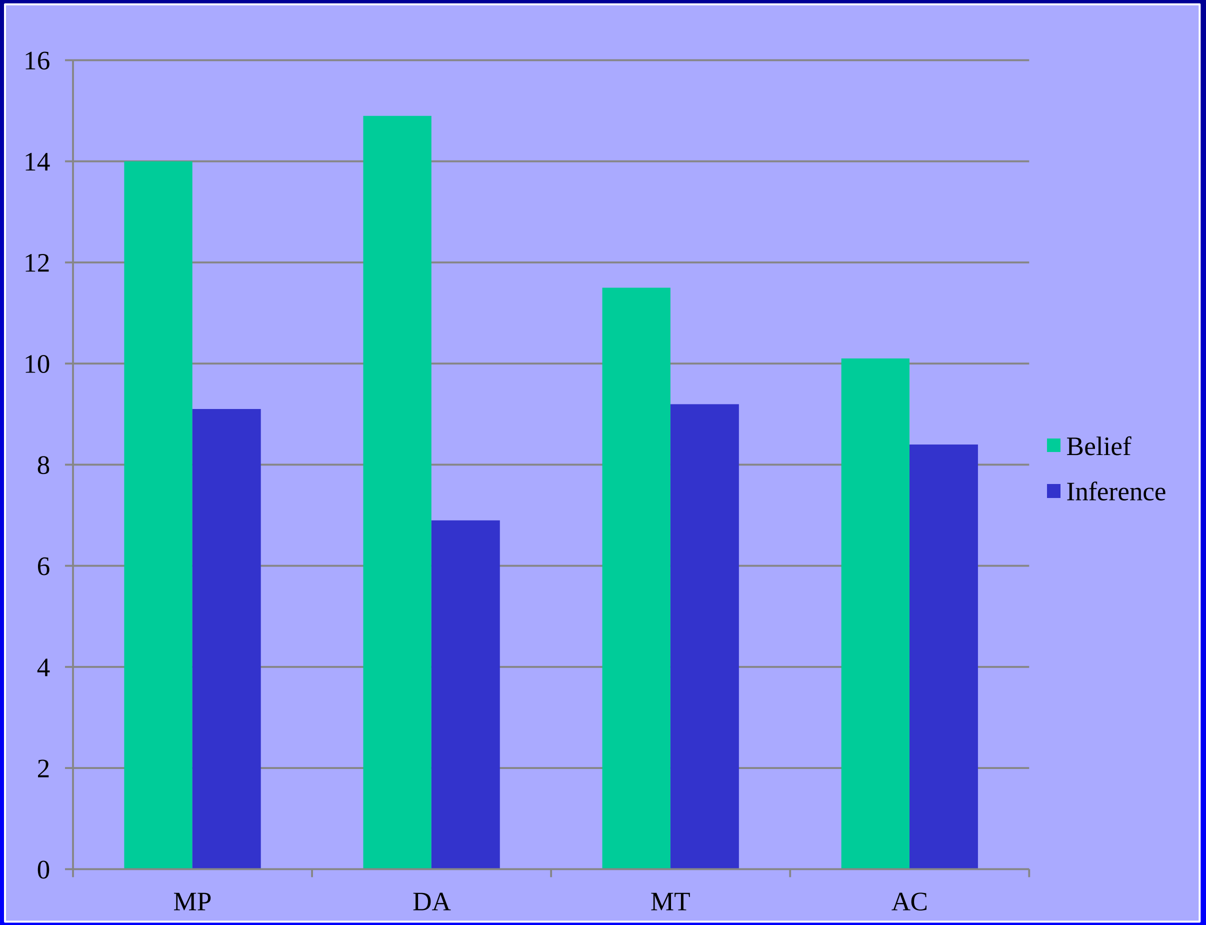
- **The four inferences - MP, DA, AC, MT - were then constructed in the analysis by comparing the relevant probability ratings from the separate tasks**

Inference group (n = 23)

For this group, the same conditionals and events were used but ratings were given to sets of three statements presented as one of the conditional inferences, e.g. AC.

GIVEN	%
– If more houses are built then the number of homeless people will decrease	__
– The number of homeless people will decrease	__
THEREFORE	
– More houses will be built	__

p-validity analysis - % violations



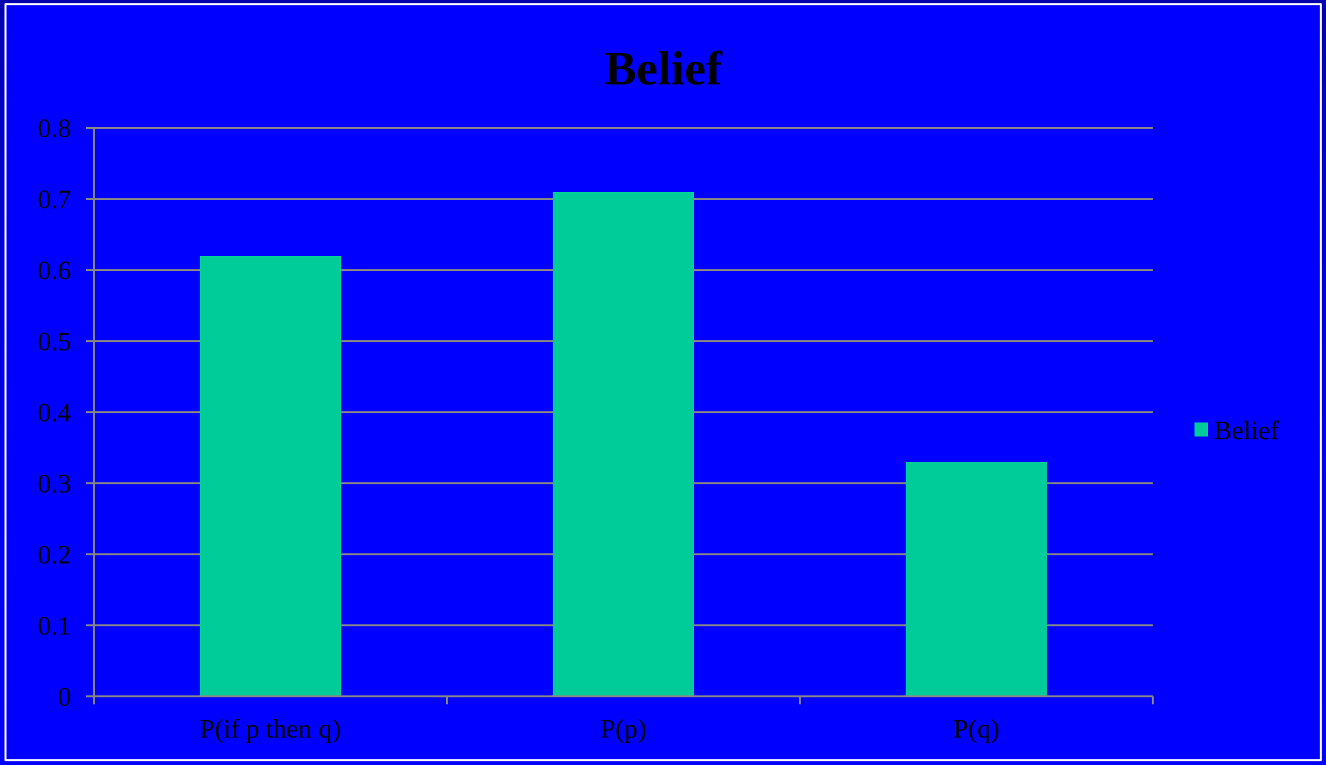
Comments on p-validity

- Only MP and MT are required normatively to confirm to p-validity.
- Violations are quite low for all inferences.
- Inference group participants show fewer violations.
- Explicit reasoning improves p-validity

Example of strong violator (MP) 1

61% Belief group

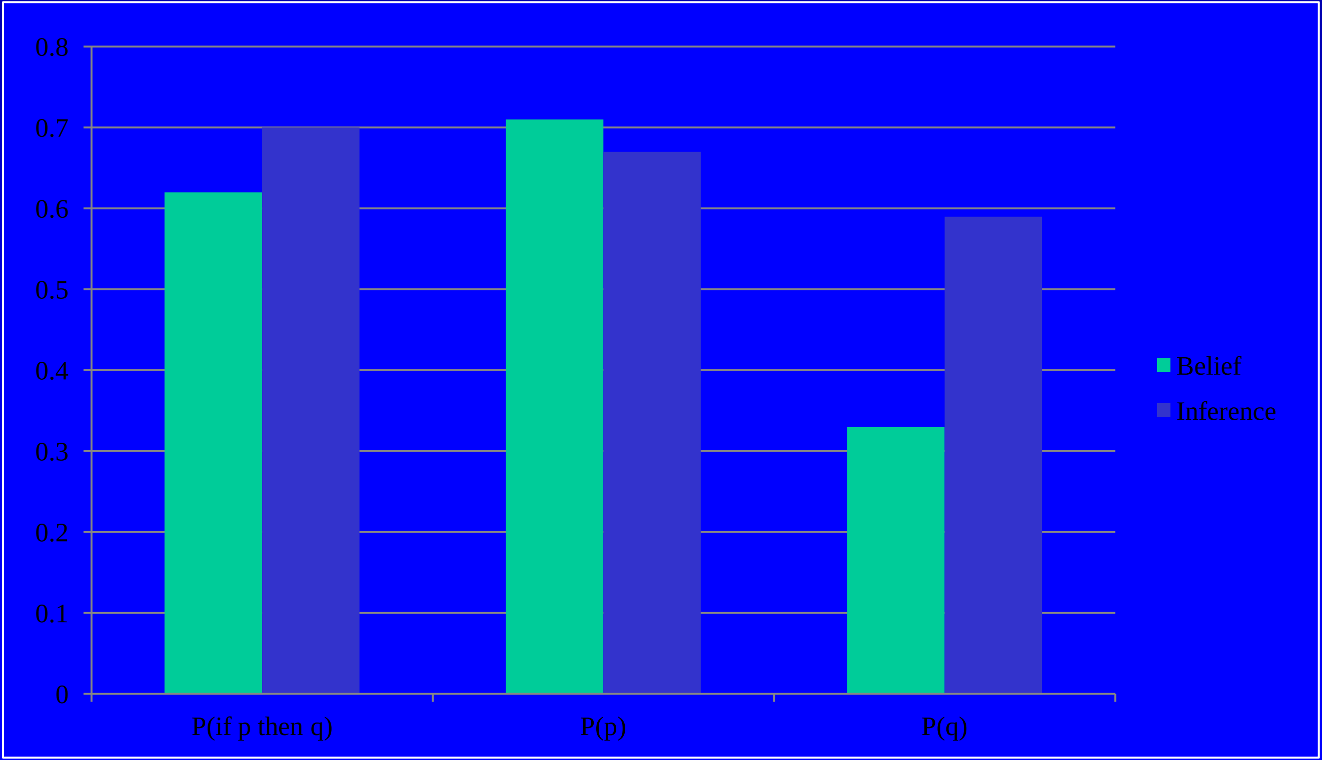
"If jungle deforestation continues, gorillas will become extinct"



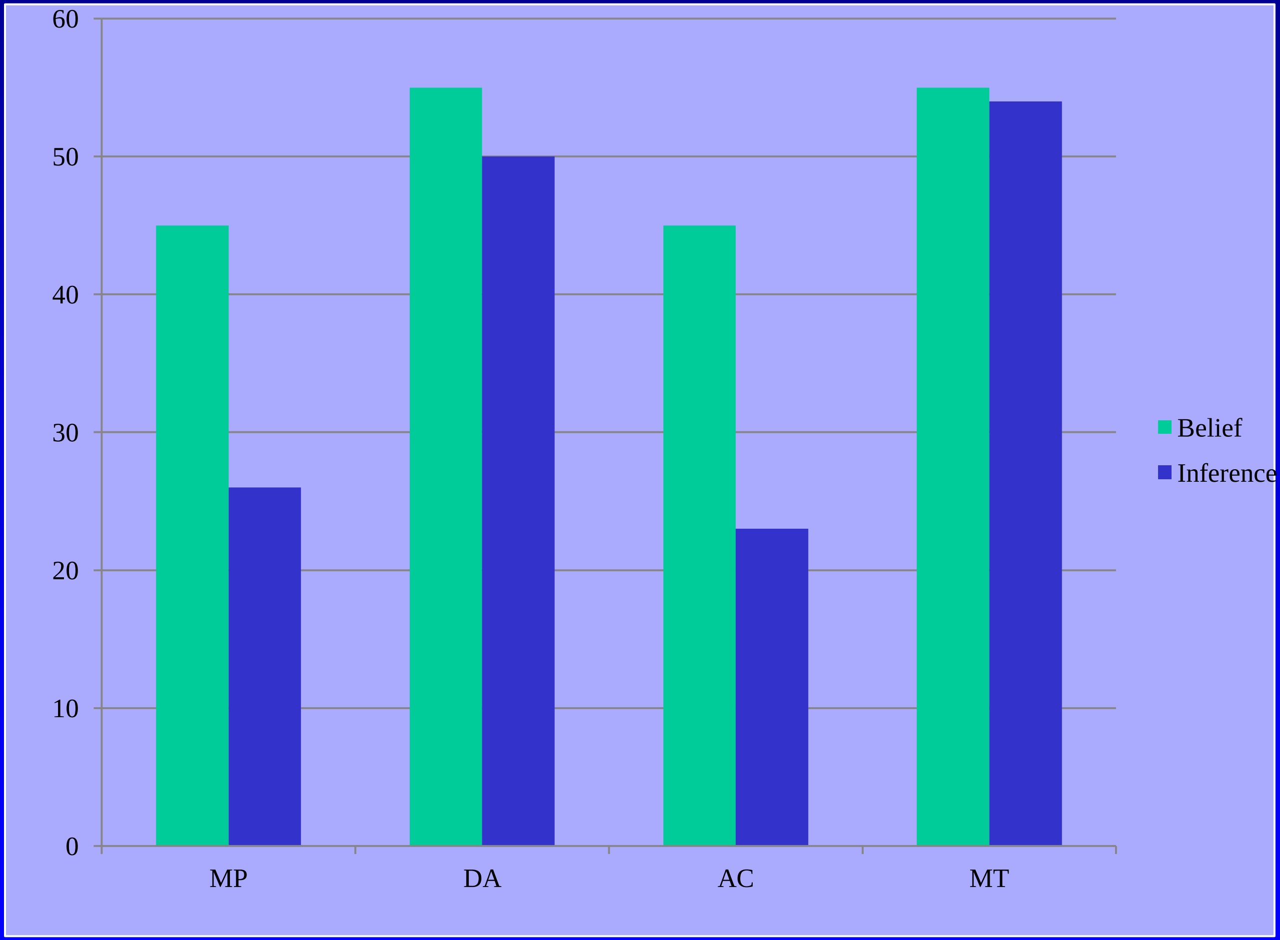
Example of strong violator (MP) 2

61% Belief group, 13% Inference group

“If jungle deforestation continues, gorillas will become extinct”



Coherence analysis - % violations



Comments on coherence

- All four inferences should conform with coherence. Violations are generally high in the Belief group, in some cases over 50%.
- However, performance is much better in the Inference group.

Evidence for dual processing

- This is a preliminary and small scale study but it is clear that an inference frame reduces violations of both p-validity and p-consistency.
- Suggest this reflects Type 2 processing which can improve consistency in our belief systems.
- Further research could introduce dual process manipulations: working memory load, speeded tasks, and cognitive ability measures.

Experimental conclusions

- **Beliefs show moderate violations of p-validity and substantial violations of coherence when conditional inferences are implicit.**
- **When explicit inferences are given there is a significant improvement in both p-validity and coherence, suggesting type 2 intervention.**

General conclusions

The old paradigm tried to study human reasoning from arbitrary assumptions in extensional and binary logic and "discovered" many "fallacies".

The new paradigm is Bayesian / probabilistic and studies inference from degrees of belief. Its account of human reasoning will be a much fairer and deeper assessment of human fallacies and rationality.

To understand human rationality, degrees of belief in the premises and conclusions of even logically valid inference must be measured and evaluated in the new paradigm.