Integrating the study of reasoning and judgment and decision making in the new psychology of reasoning

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The old paradigm: The conditional

In the old paradigm, a conditional *if p then q* was usually assumed to be equivalent to the truth functional / material conditional, *not-p or q*.

Johnson-Laird & Byrne (1991) note that *if p then q* is true when *p* and *q* are both true, and false when *p* is true and *q* is false. They then ask whether *if p then q* is true or false when *p* is false. They say (p. 7), "It can hardly be false, and so since the propositional calculus allows only truth and falsity, it must be true." Clearly, these truth conditions for *if p then q* make it equivalent to *not-p or q*.

Older paradigm conditionals: Mental model theory (Byrne & Johnson-Laird, 2009)

The indicative conditional, "if p then q", has for its "core" an initial model plus other models that can be implicit:

p q

…

or explicit when … becomes:

not-p q

not-p not-q

What the theory of mental models implies about probability judgments

There will be a tendency for people to judge that P(if p then q) is the probability of the initial model:

$$P(\text{if p then q}) = P(p \,\&\, q)$$

But people will sometimes make the implicit models explicit and judge P(if p then q) on the basis of all the models:

$$P(\text{if p then q}) = P(\text{not-p or q})$$

People "should" make this judgment (Byrne & Johnson-Laird, (2009, p. 285)

The old paradigm and decision making

Suppose that *If we buy a UK National lottery ticket, then we will win millions* is equivalent to *not-buy or win millions*. It will become more and more probable as we become more and more determined not to buy a ticket. How then can it be rational not to buy a ticket?

The old paradigm and its account of the conditional cannot be integrated with judgment and decision making.

The new paradigm: Basic points

There is a new Bayesian, or probabilistic, approach in the psychology of reasoning. Human inference is not from arbitrary assumptions, but from beliefs that are usually uncertain to some degree.

The relevant normative theory in the new approach is the *logic of partial belief* (Ramsey) / *the logic of subjective probability* or *uncertainty* (de Finetti). It goes beyond the binary and extensional logic of certainty found in the propositional calculus. Its account of human rationality is Bayesian / probabilistic.

The old and new paradigms: An example

If we buy a UK National lottery ticket, we will win millions.
We will buy a UK National lottery ticket.
Therefore, we will win millions.

In older paradigms, the premises of this instance of MP are to be assumed, in effect to be treated as certain, and the question is what necessarily follows. But the new paradigm takes account of uncertainty premises.

The new paradigm and the conditional

The natural language indicative conditional is held to be the probability conditional (Adams) or the conditional event (de Finetti). Its probability is the conditional probability:

$$P(\text{if } p \text{ then } q) = P(q|p)$$

The above is sometimes called the Equation both in philosophy and psychology (Edgington, 1995a; Oaksford & Chater, 2007, 2009) and the conditional probability hypothesis when seen as a descriptive statement. The psychological process at the basis of it in the new paradigm is generally taken to be the Ramsey test (Evans & Over, 2004).

The Equation as the conditional probability hypothesis

The conditional probability hypothesis, that P(if p then q) = P(q|p) , has been highly confirmed in recent experiments (including Evans et al., 2003; Over et al, 2007; Douven & Verbrugge, 2010; Fugard et al., 2011). Versions of the new paradigm most relevant to this talk are those of Evans & Over (2004), Oaksford & Chater (2007), Baratgin , Over, & Politser (2013), and Pfeifer & Kleiter (2010).

Introducing conditionals: The Ramsey test

Ramsey (1931): People could judge "if p then q" by "...adding p hypothetically to their stock of knowledge …" They would thus fix "...their degrees of belief in q given p…", P(q|p).

In Ramsey's original example, the two people were arguing about "if p then q", and so there could be a winner and a loser in the debate.

The basic de Finetti / "defective" truth table for *if p then q*

T = true, F = false, W = win, L = lose, V = void

p \ *q*	T	F
T	T (W)	F (L)
F	V	V

The Ramsey test and de Finetti table

The Ramsey test and de Finetti table are the pillars that support Bayesian, or probabilistic, accounts of human conditional reasoning. In such accounts, indicative conditionals, conditional bets, and the conditional probability should all be closely related to each other.

The lottery example

Recall the example:
If we buy a UK National lottery ticket, we will win millions

In the new paradigm, it no longer follows, as it does in the old paradigm, that this conditional gets more probable as it becomes more likely that we will not buy a ticket. The conditional probability - that we will win the lottery given that we buy a ticket - stays extremely low as we become less and less likely to buy a ticket.

Reasoning and decision making

- The new Bayesian paradigm can integrate the study of reasoning and decision making in psychology.

- Consider the conjunction fallacy as a simple example. This occurs when one judges that the probability of "p & q", P(p & q), is greater than the probability of q, P(q), although q clearly follows validly from "p & q".

- In the new paradigm, validity is defined directly in terms of probability. A single premise inference is valid if and only if the probability of its premise cannot be coherently greater than the probability of its conclusion.

The Linda problem

Linda is single, outspoken, and intelligent. She majored in Philosophy at university, was concerned with social justice, and was anti-nuclear (Tversky & Kahneman, 1983).

Which is more probable?

Linda is a bank teller.

Linda is a feminist and a bank teller.

The conjunction / Linda fallacy

- **Participants in experiments tend to judge P(f & t) > P(t), when they make judgments about Linda's qualities.**

- **Psychologists have long noted that judging P(f & t) > P(t) is incoherent because of the logical relation between "f & t" and t, but only with the coming of the new paradigm has account been taken of the much more general relation between probability and logical validity.**

New paradigm: Validity

P(p & q) cannot be coherently greater than P(p).

More generally, the probabilities of the premises of a valid inference cannot be coherently greater than the probability of the conclusion.

More generally and more precisely, when P(if p then q) = P(q|p), let the uncertainty of any premise or conclusion s be 1 - P(s). Then an inference is p-valid if and only if the uncertainty of its conclusion cannot be coherently greater than the sum of the uncertainties of its premises.

New paradigm: Coherence

Coherence can be seen as a generalization of the binary notion of consistency, giving us intervals for belief.

It is binary inconsistent to believe *p* & *q* but not *p*.

More generally, where the degree of belief in *p* is P(p) and where the degree of belief in *q* is P(q):

$$\min(P(p), P(q)) \geq P(p \,\&\, q) \geq \max(0, P(p) + P(q) - 1)$$

Incoherence: An conjunctive example

Linda is at the party (p) and she is enjoying herself (q) .

Let P(p) = .6 and P(q) = .6

If people claim that P(p & q) > .6, they are committing a fallacy. And they are also committing a fallacy if they claim P(p & q) < .2.

Incoherence in a set of beliefs

Linda is single, outspoken, and intelligent. She majored in Philosophy at university, was concerned with social justice, and was anti-nuclear. Rank the following in probability:

Linda is a bank teller.

Linda is a social worker.

Linda is a feminist and a bank teller.

Linda is a farmer.

Incoherence in an inference

Linda is single, outspoken, and intelligent. She majored in Philosophy at university, was concerned with social justice, and was anti-nuclear. Consider:

Linda is a feminist and a bank teller.

Therefore, Linda is a bank teller.

What probability do you assign to the above premise?

What probability do you assign to the above conclusion?

Comparing beliefs and inferences

Tversky & Kahneman (1983) found incoherence in a set of beliefs about Linda. But they did not ask whether people tend to be incoherent in their inferences about Linda. And no one else has done this either.

Does explicit inference or reasoning improve the coherence of people's judgments?

No one has yet studied this question. To address it we must take the new approach to the psychology of reasoning and begin to integrate it with judgment and decision making.

What is coherence?

Have so far used "coherence" in de Finetti's sense and that of most basic research in probability theory. That is, beliefs are coherent if a Dutch book cannot be made against them. Let us call this "narrow coherence".

The ordinary understanding of coherence is "wider" and has to do with how beliefs "hang together". Do the beliefs support each other, or do they form an explanatory scheme in some sense?

The "wide" notion has something to do with inferences that support the body of beliefs.

What is wide coherence?

Will show that people have more "narrow" coherence when they make explicit inferences among their beliefs.

Is it then simply a matter of fact that people have intuitive "wide" coherence when they are "narrow" coherent, due to fact that they only the latter when they connect their beliefs with inferences?

Or do people only have "wide" coherence when their beliefs are related by strong or forceful inferences? Or when their inferences not only have these qualities but something else?

Strong and forceful inferences

An inference is strong to the extent that its conclusion is probable given degrees of belief in its premises, and it is forceful to the extent that it leads to a large change in the degree of belief in its conclusion.

Many inferences seen as "fallacies" by the old paradigm are viewed as strong or forceful inferences in many contexts in a Bayesian analysis, and perhaps these inferences are what is important for the intuitive "wide" coherence?

Narrow incoherence and the conditional

Have defined this coherence for conjunction by giving an interval.

This coherence can also be defined for the conditional and conditional inferences by giving intervals. The intervals for conditional inferences can be derived from the Equation:

$$P(\text{if } p \text{ then } q) = P(q|p)$$

"Wide" coherence will have to be addressed in future work.

Conditional inferences

If Linda was at the party, she drank too much.

Modus Ponens (MP) Valid

She was at the party; so she drank too much.

Modus Tollens (MT) Valid

She did not drink too much; so she was not at the party.

Affirmation of the Consequent (AC) Invalid

She drank too much; so she was at the party.

Denial of the Antecedent (DA) Invalid

She was not at the party; so she did not drink too much.

Recall generalizing p-validity

The probabilities of the premises of a valid inference cannot be coherently greater than the probability of the conclusion.

More generally and more precisely, let the uncertainty of any premise or conclusion *s* be 1 - P(s). Then an inference is p-valid if (and only if) the uncertainty of its conclusion cannot be coherently greater than the sum of the uncertainties of its premises.

Note that the definition of p-validity does not presuppose that P(if p then q) = P(q|p).

Coherence and more than one premise

No extension of coherence is required for more than one premises. A set of degrees of belief is coherent if and only a Dutch book cannot the made against it.

Supposing that P(if p then q) = P(q|p), intervals can be derived for conditional inferences with more than one premise. The set of degrees of belief in the premises and the conclusion will be coherent if and only if the degree of belief in the conclusion is within the interval.

Consider an example of MP

If Linda goes to the party (p), then she will drink too much (q). She will go to the party. Therefore, she will drink too much.

Suppose our degree of belief in the conditional, major premise above is $P(q|p) = .9$.

Suppose our degree of belief in the minor premise is $P(p) = .8$.

And suppose our degree of belief in the conclusion is $P(q) = .95$.

Are we coherent in our beliefs?

The central example of a two premise inference: MP

If Linda goes to the party (p), then she will drink too much (q). She will go to the party. Therefore, she will drink too much.

Where P(q|p) = .9 and P(p) = .8, P(q) should be by total probability:

$$P(q) = P(p)P(q|p) + P(\text{not-}p)P(q|\text{not-}p) = .72 + .2P(q|\text{not-}p)$$

Clearly, MP is p-valid. Moreover, P(q|not-p) will be 0 at the minimum and 1 at the maximum, P(q) should fall in the interval [.72 , .92] for coherence.

If people claim that $P(q) < .72$ or $P(q) > .92$, they are committing a fallacy. This is like the Linda fallacy at a general level. The old paradigm cannot even express the idea that confidence in the conclusion of MP can be too high.

Conforming to p-validity and the interval by chance

If Linda goes to the party (p), she will drink too much (q). She will go to the party. Therefore, she will drink too much.

Where $P(q|p) = .9$ and $P(p) = .8$, $P(q)$ should be by total probability:

$$P(q) = P(p)P(q|p) + P(\text{not-}p)P(q|\text{not-}p) = .72 + .2P(q|\text{not-}p)$$

The sum of the uncertainties of the premises is $.1 + .2 = .3$. To conform to p-validity, we must judge $P(q) \geq .7$, as $1 - .3 = .7$. The chance that we will be correct by guessing is .3.

To be coherent, we must judge $.72 \leq P(q) \leq .92$. The chance that we will be coherent by guessing is $.92 - .72 = .2$.

Do people respect p-validity and coherence in their conditional beliefs and inferences?

- One way to get an answer is simply to ask people to rate their beliefs in *if p then q*, *p*, *q*, *not-p*, for a particular set of realistic conditionals.

- We can then compare the probabilities assigned for p-validity and coherence.

- This measures <u>implicit</u> p-validity and coherence in their beliefs and is a very strong test of the Bayesian requirement that subjective probabilities conform with the probability theory.

Explicit reasoning and dual process theory

- **Dual process theory holds that many degrees of belief are formed by implicit heuristic, type 1, processes, but explicit reasoning is a type 2 process that allows the use of logical or other rules in working memory.**

- **When beliefs are grouped together to indicate their inferential structure, type 2 processes can use logical and coherence relations in probability judgments.**

- **Such grouping tests whether p-validity and coherence can be achieved with explicit reasoning effort.**

EXPERIMENT

Jonathan Evans, Valerie Thompson, & David Over

- **Have run a small study as a first step, and constructed 48 realistic conditional sentences expressing causal or temporal relations about events in the near future, e.g. If more houses are built, then the number of homeless people will decrease/increase.**

- **The study was run in Saskatoon and statements contextualised for Canadian students.**

Belief group (n = 23)

- Rated one list of 48 conditionals, *if p then q*, assigning probabilities to represent beliefs in randomized order.

- Rated a second event list containing both affirmative and negative statements representing *p*, *not-p*, *q*
- and *not-q* to make minor premises and conclusions for inferences.

- The four inferences - **MP**, **DA**, **AC**, **MT** - were then constructed in the analysis by comparing the relevant probability ratings from the separate tasks

Inference group (n = 23)

For this group, the same conditionals and events were used but ratings were given to sets of three statements presented as one of the conditional inferences, e.g. AC.

GIVEN **%**

- **If more houses are built then the number of homeless people will decrease** ___
- **The number of homeless people will decrease** ___

THEREFORE

- **More houses will be built** ___

p-validity analysis - % violations

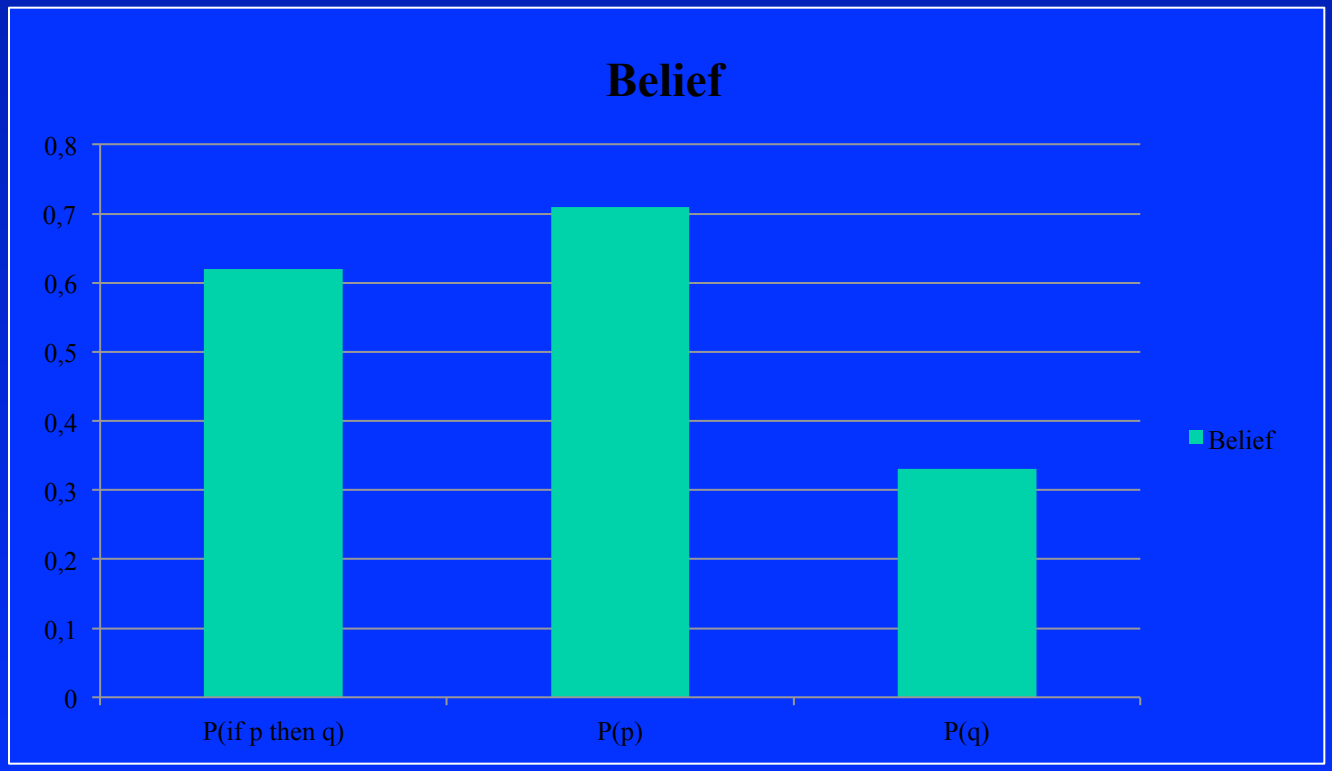
Comments on p-validity

- Only MP and MT are required normatively to confirm to p-validity.
- Violations are quite low for all inferences.
- Inference group participants show fewer violations.
- Explicit reasoning improves p-validity, but correct for being right by "chance".

Example of strong violator (MP) 1

61% Belief group

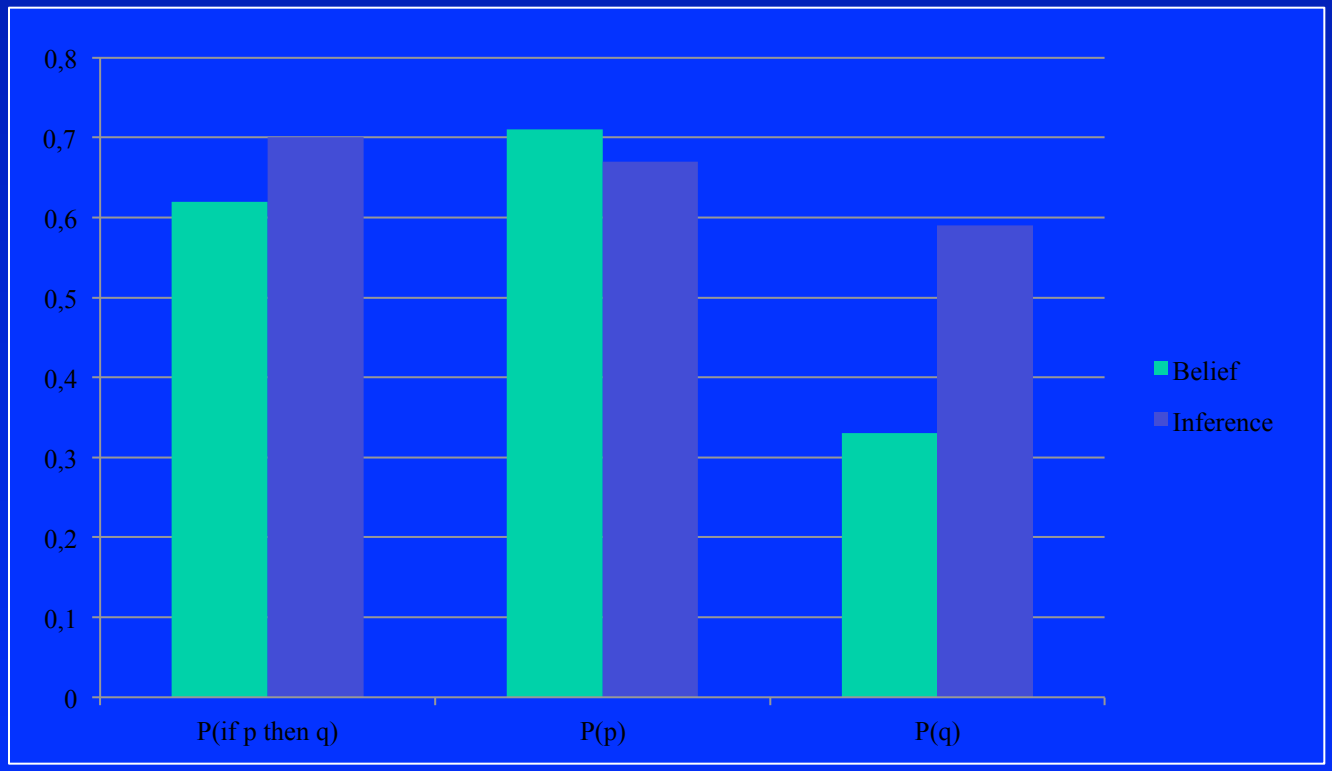
"If jungle deforestation continues, gorillas will become extinct"



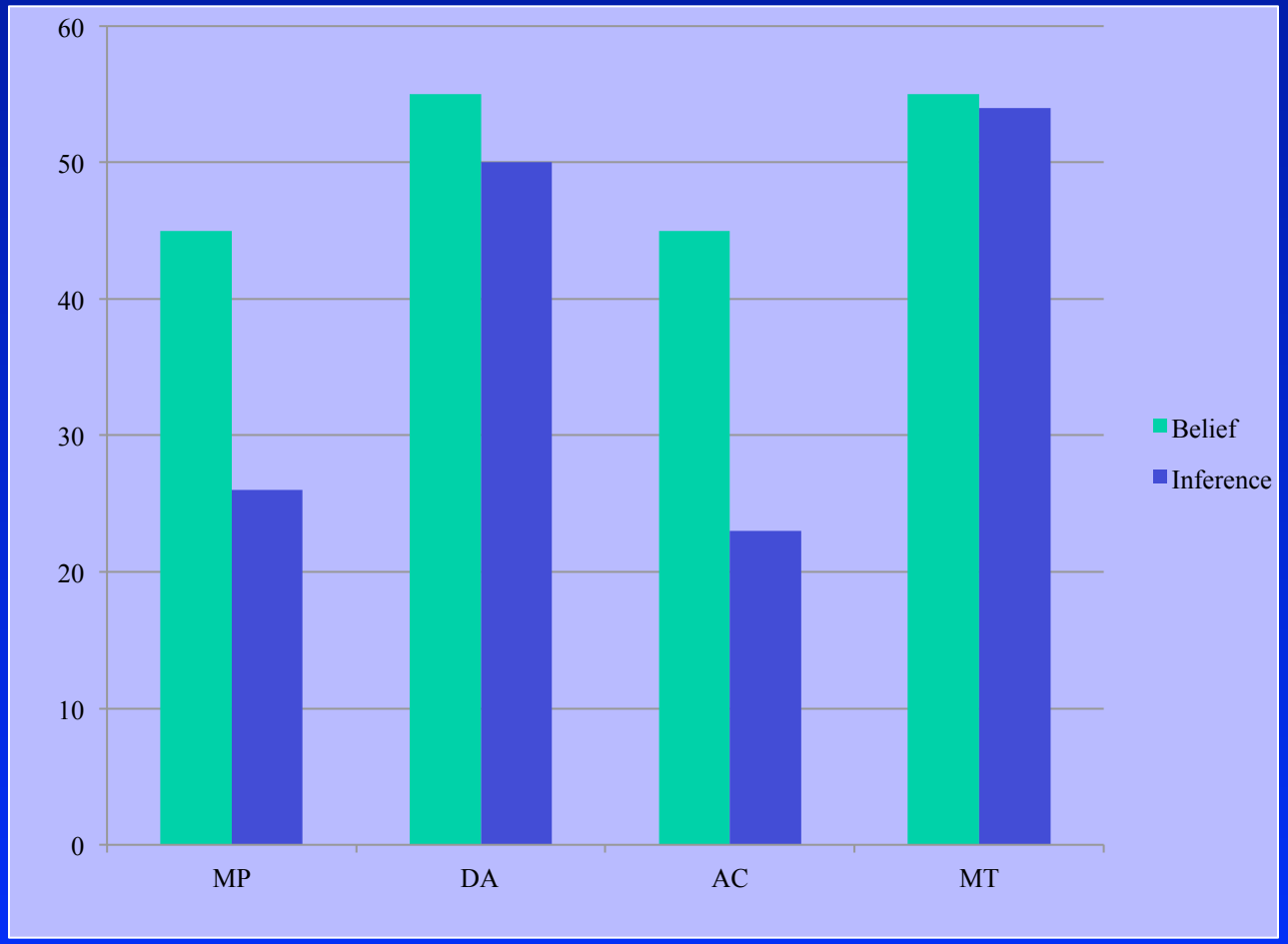
Example of strong violator (MP) 2

61% Belief group, 13% Inference group

"If jungle deforestation continues, gorillas will become extinct"



Coherence analysis - % violations



Comments on coherence

- All four inferences should conform with coherence. Violations are generally high in the Belief group, in some cases over 50%.

- Performance is better in the Inference group, but correct here also for being right by "chance".

Experimental conclusions

- **Beliefs show moderate violations of p-validity and substantial violations of coherence when conditional inferences are implicit.**

- **When explicit inferences are given there is a significant improvement in both p-validity and coherence for MP inferences at least, implying type 2 intervention.**

Evidence for dual processing

- This is a preliminary and small scale study but it seems to confirm that an inference frame, and especially MP, can reduce violations of both p-validity and coherence. See also research with Christoph and Henrik.

- Some suggestion that Type 2 processing can improve consistency in people's beliefs .

- Could introduce dual process manipulations - working memory load, speeded tasks, cognitive ability measures - in future research.

Additional future research

- **Focus on premises with relatively high probability, making it is harder to be right by "chance".**

- **Study a wider range of inference forms and integrate with studies in judgment and decision making.**

- **Conditionalization, or diachronic MP, is fundamental to Bayesian belief updating and revision, and it is certainly of interest that MP appears to have the greatest effect on p-validity and coherence. Much more research should be done on belief updating and revision in the new paradigm.**

General conclusions

The old paradigm tried to study human reasoning from arbitrary assumptions in extensional and binary logic and "discovered" many "fallacies".

The new paradigm is Bayesian / probabilistic and studies inference from degrees of belief. Its account of human reasoning will be a much fairer and deeper assessment of human fallacies and rationality.

To understand human rationality, degrees of belief in the premises and conclusions of even logically valid inference must be measured and evaluated in the new paradigm.